AN EXPERIMENTAL STUDY OF RELAY SERVO OPTIMIZATION USING A SERIES MOTOR

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Commander, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
ELECTRICAL ENGINEERING

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ABSTRACT

Use of the series motor in servomechanisms has been somewhat limited in the past, perhaps because other types permit linear techniques to be used in system design.

It is the purpose of this paper to show by experimental means that the non-linearity of the series motor in fact provides the necessary characteristic to permit relay servo optimization by the use of switching criteria based simply on error and error rate.

To reduce the probability that such a desirable characteristic might hold only for the specific motor tested, three split-field series motors of varying sizes were used in the investigation. Various motor supply voltages were used, including alternating current. Dead beat response to any size step displacement input was found possible in all cases using only error and error rate control as switching criteria.

Response to sinusoidal and ramp inputs was also investigated.

The advice and assistance of Dr. George J. Thaler of the U. S. Naval Postgraduate School is most sincerely appreciated.

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TABLE OF SYMBOLS

A, B	Gain constants
E	Error, degrees
$\dot{E}$	Error rate, degrees/sec.
f_{eq}	Coefficient of viscous friction referred to the motor shaft
i_a	Armature current
J_{eq}	Moment of inertia of the motor-load referred to the motor shaft
k, k_1, k_2	Proportionality constants
L_M	Motor inductance, including both armature and field
M	Magnitude ratio, frequency response
R_M	Motor resistance, including both armature and field
V	Applied voltage
V_R	Voltage applied to the relay coil
Θ_C	Controlled angular output position, degrees
Θ_R	Reference angular position, degrees
Θ_M	Motor shaft position, degrees
$\dot{\Theta}$	Angular velocity, deg./sec.
$\ddot{\Theta}$	Angular acceleration, deg./sec^2
Φ	Magnetic flux, motor field
ϕ	Phase angle, frequency response
ω_i	Input frequency, radians/sec.
ω_R	Ramp input velocity, deg./sec.

INTRODUCTION

The relay or contactor servomechanism fills an important place in the field of automatic control systems. It's principal advantages are it's light weight, low cost, fast response, and overall simplicity. The relay itself provides a high power gain, permitting control of large loads with a small control amplifier. It's principal disadvantages are sensitivity to vibration and shock, and contact arcing which may become a difficult problem at high altitudes, particularly if the motor supply voltage is d. c. and the load requirements are large.

With the recent advent of the solid state thyratron, it is believed that the non-linear servomechanism will lose it's primary disadvantage, i. e., relay contact arcing, and will take on an increasing importance in the field of automatic control. In addition to the elimination of contact arcing, the silicon controlled rectifier used in place of the electromechanical relay in the non-linear servomechanism shows promise of providing the design engineer with other important features, namely; a nearly ideal switching characteristic, greatly improved reliability, low forward voltage drop, "deionization" time on the order of a few microseconds, and surprisingly low size and weight.

To obtain optimum response to step displacement inputs, a relay servomechanism must have two phases of operation, namely; an acceleration period during which time full forward voltage is applied to the driving motor, then a deceleration period wherein full reverse voltage is applied. If the reversal of input to the motor is timed properly, the system terminates it's motion at the commanded position in the shortest possible time with error and error rate reaching zero simultaneously.

True optimum response would of course be possible only if the controlling relay possessed ideal switching characteristics with no hysteresis or dead zone. While such a relay is not feasible, it is possible to reduce the dead zone to quite respectable values and thus approach the optimum system. Dead zones of less than 0.1 degree were obtained in the course of the investigation.

Harris, McDonald, and Thaler (1) have developed means of quasi-optimization of relay servomechanisms using discontinuous damping in the dead zone. These studies were made using either the two phase motor or the shunt field d. c. motor.

Goslow (2) found the series motor to possess near optimum response to step displacement inputs while controlled by an electronic relay utilizing FG-81A thyratrons in a relay-rectifier control unit. This unit provided a very narrow dead zone, however the switching characteristics were not vertical as is the case with an electromechanical relay. It remained to determine whether dead beat response could be achieved with the series motor while using an electromechanical relay for motor control.

The requirement for dead beat operation of a relay servomechanism is that the relay switching line coincide with the phase plane deceleration trajectory of the system. If this deceleration trajectory can be made to approximate a straight line with slope independent of initial velocity conditions, then the locus of relay switching points on the phase plane becomes a straight line for dead beat operation. A straight relay switching line in turn permits the use of a linear switching computer, the switching criteria being a function only of error and error rate.

Deceleration trajectories of two phase motors and shunt field motors possess considerable curvature, thereby requiring dual mode operation or non-linear switching computers for control. The series motor, on the other hand, decelerates much more rapidly and the straight line approximation of the phase plane deceleration trajectory becomes more nearly valid. This faster deceleration comes about as a result of the current squared torque characteristic of the machine. The torque developed is proportional to the product of the armature current and the magnetic flux. If the motor field is operated below saturation and the magnetization curve is considered linear, the magnetic flux is then proportional to the field current. Since the field and armature currents are identical in the series motor, the developed torque becomes proportional to the square of the current. At the time of relay reversal, the field is reversed while the motor has some finite velocity. The result is a back electromagnetic force reversal which now adds to the applied voltage, causing a very high current to flow during the decelerating period. With torque nearly proportional to the square of this current, the deceleration rate becomes quite high on relay reversal.

The phase plane deceleration trajectories of the three motors tested, while having some curvature, were found to be sufficiently close to a straight line as to permit dead beat operation for any size step displacement input through the use of a small amount of error rate feedback. This dead beat response was obtained using a relay dead zone as small as one degree. Step displacement inputs as small as 1.6 degrees and as large as 296 degrees were applied while using the same

linear relay switching lines with near optimum response in all cases.

For dead zones smaller than one degree, the straight line approximation of the deceleration trajectory was no longer sufficiently valid and the response was no longer dead beat. Dead zones as small as three minutes of arc were used during the course of the investigation. With very small dead zones the deceleration trajectory curvature caused the system either to overshoot a small amount (on the order of 0.1 deg.), or to step forward once or twice during the latter part of the deceleration phase. Either type of response could be selected by rotation of the relay switching lines. Early relay reversal resulted in the stepping operation while late reversal caused an overshoot.

EXPERIMENTAL SYSTEM

A schematic diagram of the experimental system is shown in Fig. 1. An operational amplifier from a Boeing Model 6568 Electronic Analog Computer was used as a summer and amplifier for operation of the relay. Relay dead zone width was readily adjusted by varying the amplifier gain.

The relay used was a Sigma Model 6FX4C 5000GD-SIL. This was a double-throw polarized type and provided full motor supply voltage to the forward field of the split-field series motor for positive inputs, and to the reverse field for negative inputs. The relay was adjusted to pull in at 6.1 volts in the forward direction and at -6.0 volts in the reverse direction. Forward drop out occurred at 1.2 volts and reverse drop out at -1.4 volts.

Three split field series motors were used in the investigation:

1. Oster series reversible, type A-21E-12R, rated at 28 volts d. c. and 0.4 amps. A reduction gear ratio of 236:1 was used between the motor and the output shaft. Tests were conducted using motor supply voltages of 24 volts d. c., 30 volts d. c., 28 volts RMS 60 cps., and 80 volts RMS 400 cps.

2. Electric Indicator split field series type FD-37 rated at 115 volts d. c., 3600 RPM, 1/125 HP. Reduction gearing was again 236:1. Tests on this motor were conducted using supply voltages of 60 volts d. c. and 83 volts RMS 60 cps.

3. Marathon Electric type DS, 1/8 HP shunt motor modified by the addition of a split series field. Reduction gear ratio was 411:1.

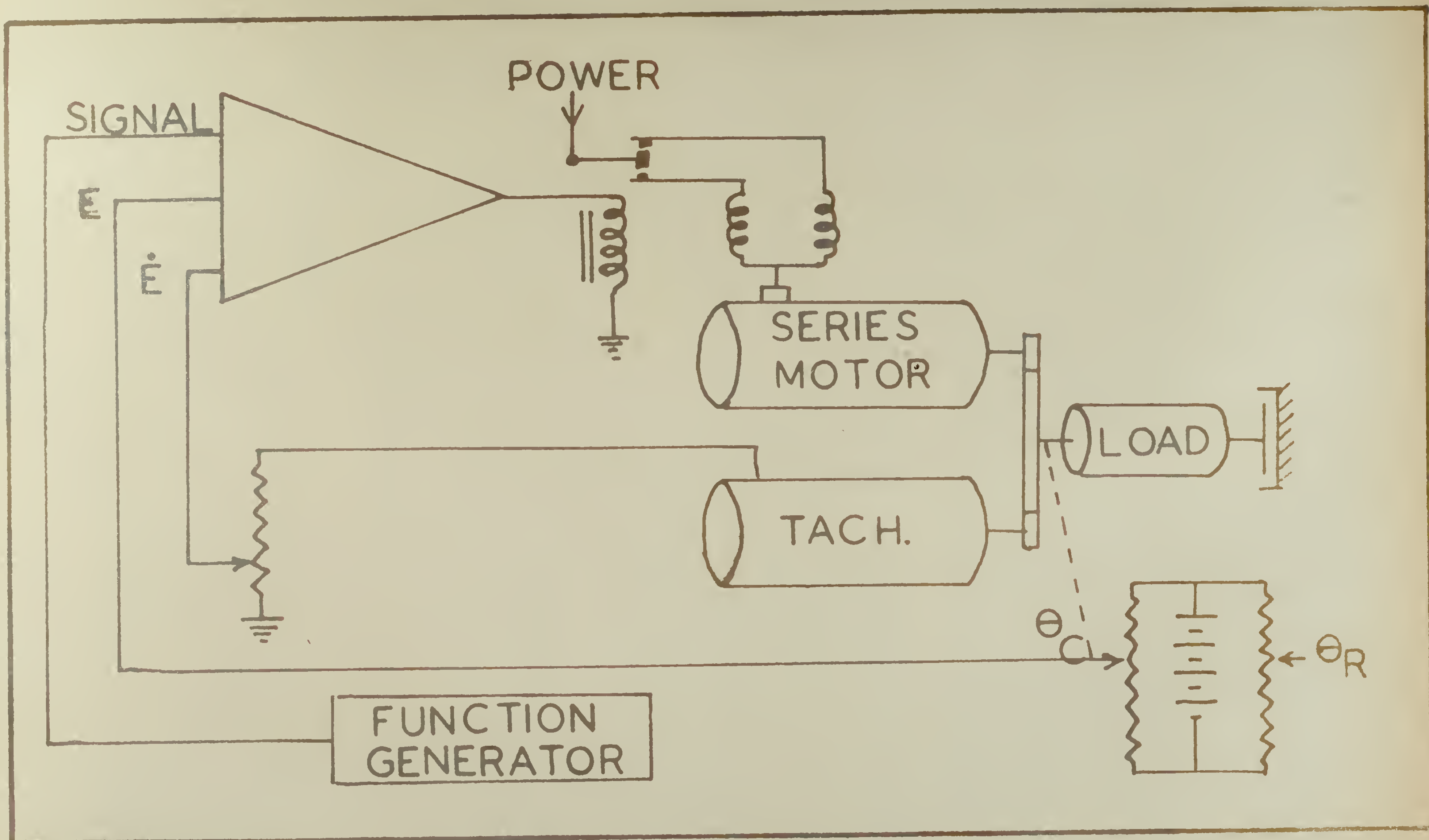

Fig. 1 SCHEMATIC DIAGRAM OF EXPERIMENTAL SYSTEM

Tests on this motor were conducted using 36 volts d. c.

Fig. 2 shows a comparison of the system response to a step displacement input for the three motors tested. Fig. 3 shows a comparison of the transient response of the Oster motor when operating on 24 volts d. c. and 28 volts 60 cps. Note that the responses are consistently dead beat, and quite similar. The use of alternating current slows down the response as would be expected since the motor is a d. c. machine. Similar curves showing the transient response of the Elinco motor operating on 60 volts d. c. and 83 volts RMS 60 cycle a. c. are shown in Fig. 4.

The error detector consisted of two potentiometers, the reference potentiometer remaining stationary while the output potentiometer was driven by the output shaft. In order to obtain maximum sensitivity the error detector was operated near it's maximum power dissipation limit. A supply voltage of 141 volts d. c. was chosen, giving 0.4 volts per degree sensitivity. Error signal was obtained between the two potentiometer sliders in a null-balance fashion.

The derivative error feedback signal was obtained from a permanent magnet tachometer whose output was a linear function of shaft velocity. A plot of tachometer output versus velocity is shown in Fig. 5. The slope of the plot is the tachometer constant:

$$K_{tach} = 0.585 \frac{V}{\text{deg./sec.}}$$

Position and velocity data were recorded by means of a Brush Electronics Company type BL-530 Dual Channel D. C. Amplifier driving a model BL-202 dual pen Brush Recorder. Simultaneous recording of output shaft position obtained from the error detector and velocity obtained

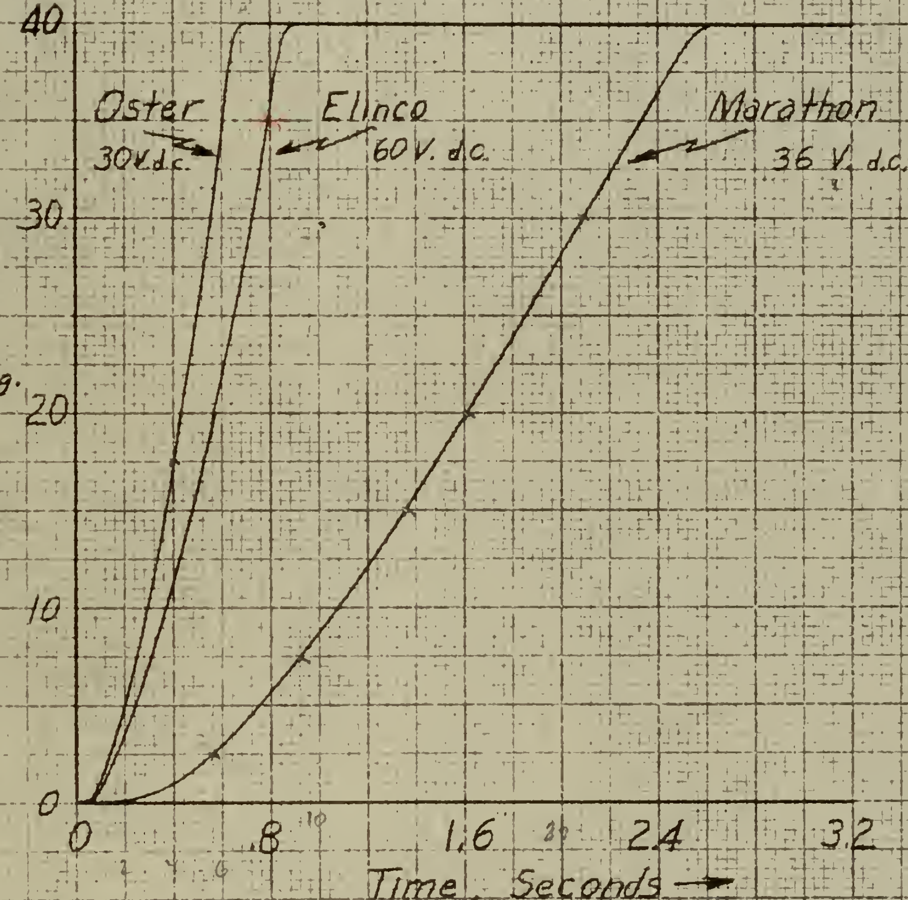

Fig. 2 COMPARISON OF SYSTEM TRANSIENT RESPONSE FOR THE THREE MOTORS TESTED

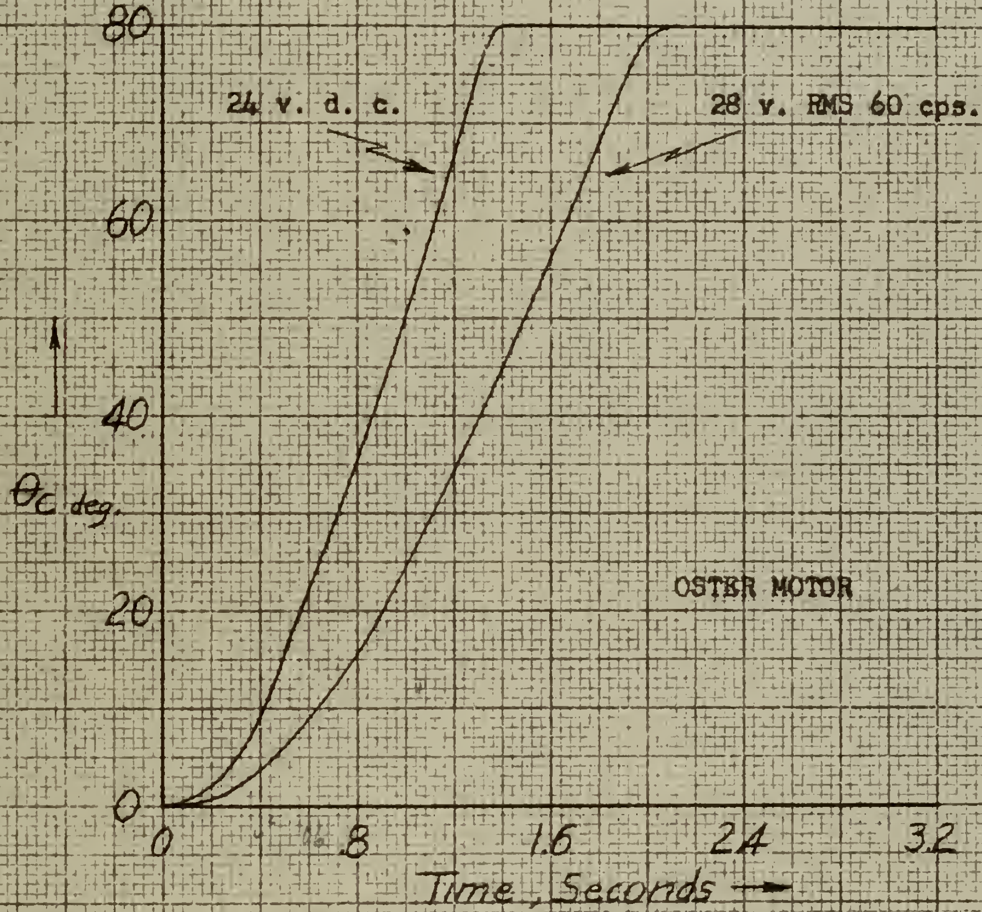

Fig. 3 COMPARISON OF TRANSIENT RESPONSE
FOR D. C. AND A. C. OPERATION

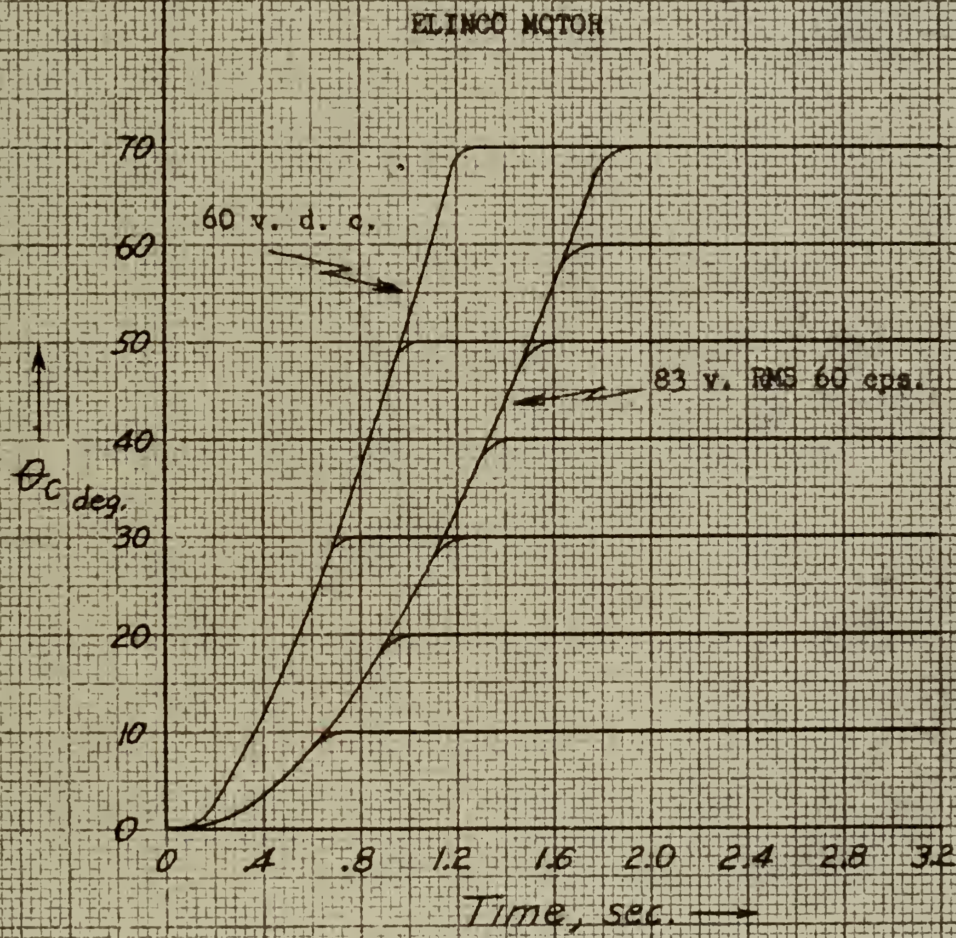

Fig. 4 COMPARISON OF TRANSIENT RESPONSE
FOR D. C. AND A. C. OPERATION

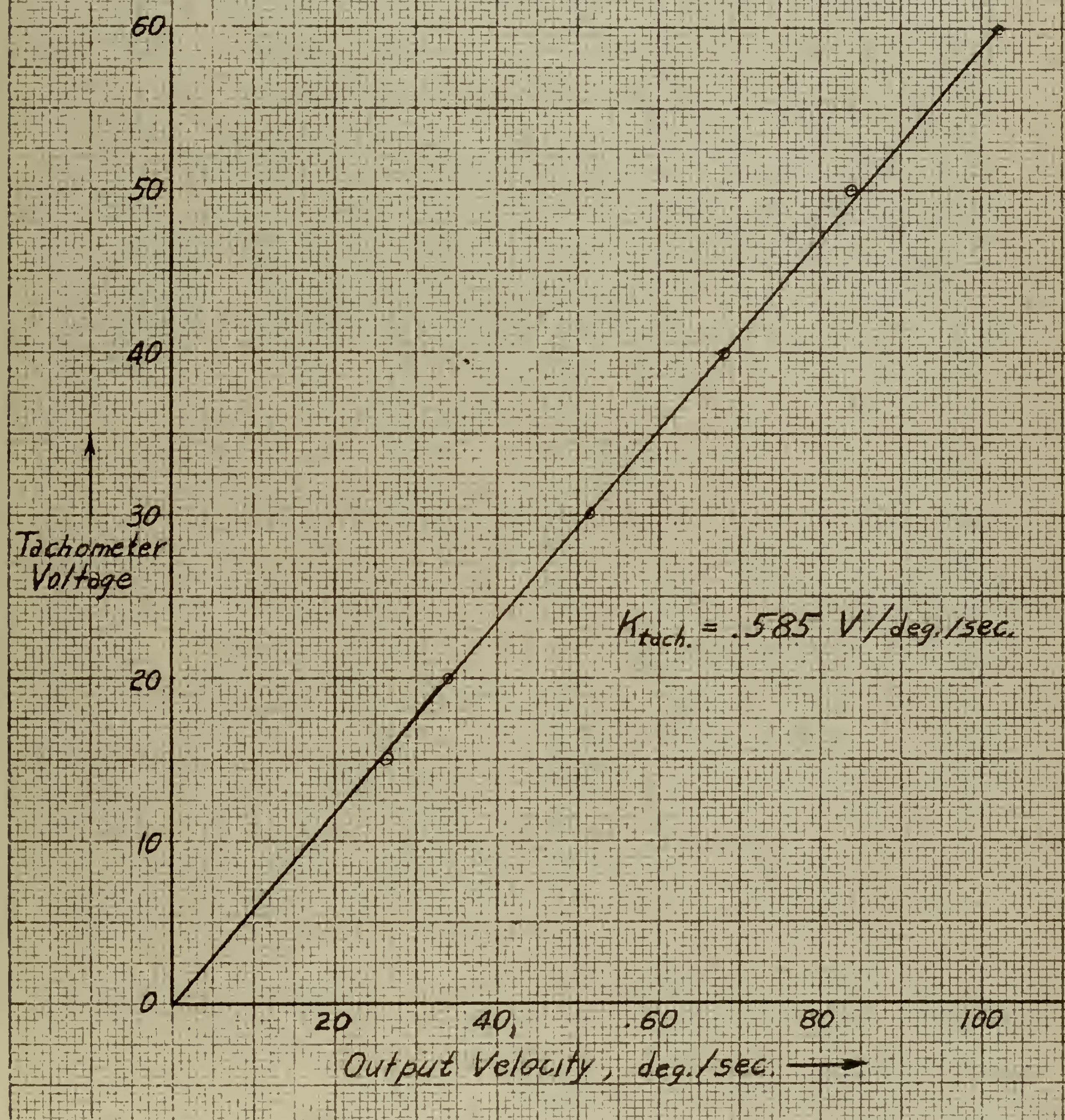

Fig. 5 TACHOMETER CALIBRATION

from the tachometer provided data which were then replotted on the E versus $\dot{E}$ phase plane.

Tests were conducted using various width dead zones by varying the amplifier gain. Dead zones as small as 3 minutes of arc were obtained on some of the tests of the Oster motor. An amplifier gain of 30 provided relay switching characteristics as shown in Fig. 6.

A Hewlett Packard model 202A Low Frequency Function Generator was used to supply the desired step displacement, step velocity, and sinusoidal inputs to the switching computer.

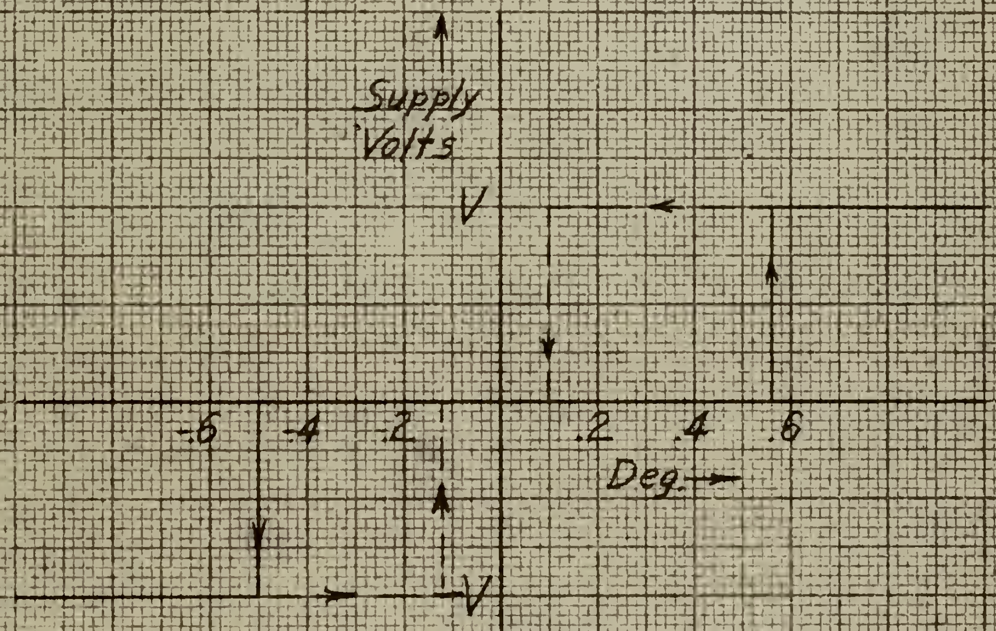

Fig. 6 RELAY SWITCHING CHARACTERISTICS
FOR AN AMPLIFIER GAIN OF 30

At the motor shaft a dynamic equilibrium must be maintained, thus:

$$J_{eq} \ddot{\Theta}_M + f_{eq} \dot{\Theta}_M = k \Phi i_a \tag{1}$$

but for the series motor:

$$\Phi = k_1 i_a \tag{2}$$

therefore:

$$J_{eq} \ddot{\Theta}_M + f_{eq} \dot{\Theta}_M = k k_1 i_a^2 \tag{3}$$

Applying Kirchoff's law to the armature circuit for a constant applied voltage to the motor:

$$V = i_a R_M + \dot{i}_a L_M + k_2 \Phi \dot{\Theta}_M$$

$$= i_a R_M + \dot{i}_a L_M + k_1 k_2 i_a \dot{\Theta}_M \tag{4}$$

Equations3 and 4 are simultaneous nonlinear differential equations defining the performance of the motor-load combination. An analytic solution to these equations is not available. Neither is a convenient phase space construction. Computer solutions might have been obtained, but it was decided that experimental solutions obtained by testing actual series motors would be more valuable, since no approximation would be necessary. The solutions are implicit in the phase plane trajectories which appear in the following section and a composite solution for the

effect of sudden voltage reversal at various initial velocity conditions is given by the phase trajectory of Fig. 7.

The relay was controlled by a switching computer consisting of a simple summing amplifier into which was fed the error signal and the output velocity signal obtained from a tachometer. The tachometer feedback was negative, and the equation of the voltage applied to the relay coil was:

$$V_R = A\,E - B\,\dot{\theta}_C \tag{5}$$

For step displacement inputs:

$$\dot{\theta}_C = -\dot{E} \tag{6}$$

therefore

$$V_R = A\,E + B\,\dot{E} \tag{7}$$

Equation 7 is the equation of the switching curves and these plot as straight lines on the E versus $\dot{E}$ phase plane. There are four such lines, one for each relay pull-in voltage and one for each drop out voltage. On the phase portraits shown in the following section, the scales are such that a single straight line through the origin will cover the entire family of relay switching curves when operating with very narrow dead zones.

TRANSIENT RESPONSE

1. Characteristics of Deceleration Trajectories.

An explanation for the consistency of the near-optimum transient response may be found in the deceleration trajectories of the series motor. These were obtained by letting the system accelerate to any desired velocity, then reversing the input. The transient variation of position and velocity were recorded and replotted on the phase plane. If a number of these trajectories are plotted (starting with different initial velocities.) and superimposed, the composite curve is as shown in Fig. 7. The shape of the individual curves is essentially the same, slight variations being readily attributable to experimental error. The important feature of this composite curve is that the major portion is nearly a straight line. This feature allows the use of a linear computer for switching and explains the reason for near-optimum response to any size step displacement input.

2. System Response to Error Control.

The response of the system to a step displacement input using only error control is shown in the time domain in Fig. 8 and on the phase plane in Fig. 9. Operation here is with the Oster motor using 24 volts d. c. and with a relay dead zone of 0.66 deg. The relay switching line for this case is the vertical or $\dot{E}$ axis, assuming an ideal relay. The effect of the finite dead zone, hysteresis, and time lag of the actual relay is to rotate the switching line clockwise in the same manner as would occur if an integrating network or phase lag compensator were used. The result is an overshoot followed by a limit

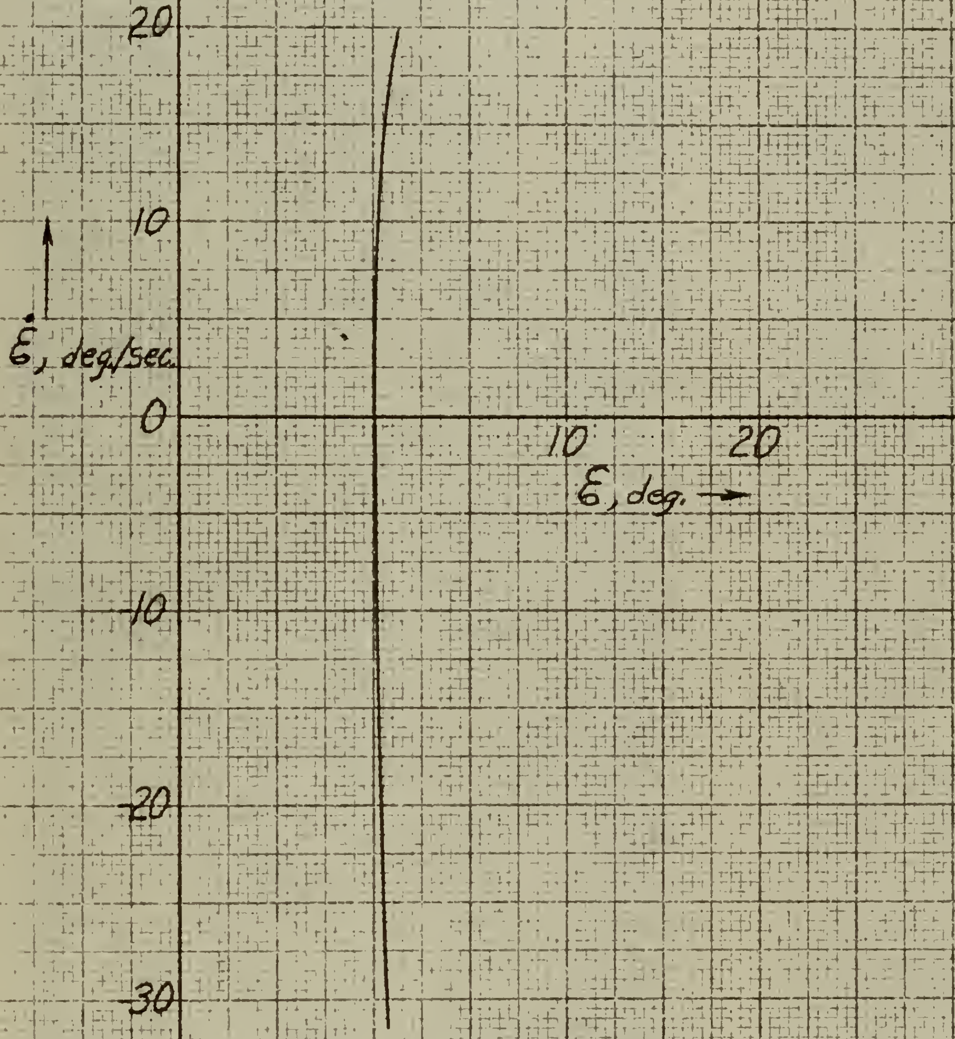

Fig. 7 COMPOSITE CURVE OF DECELERATION TRAJECTORIES

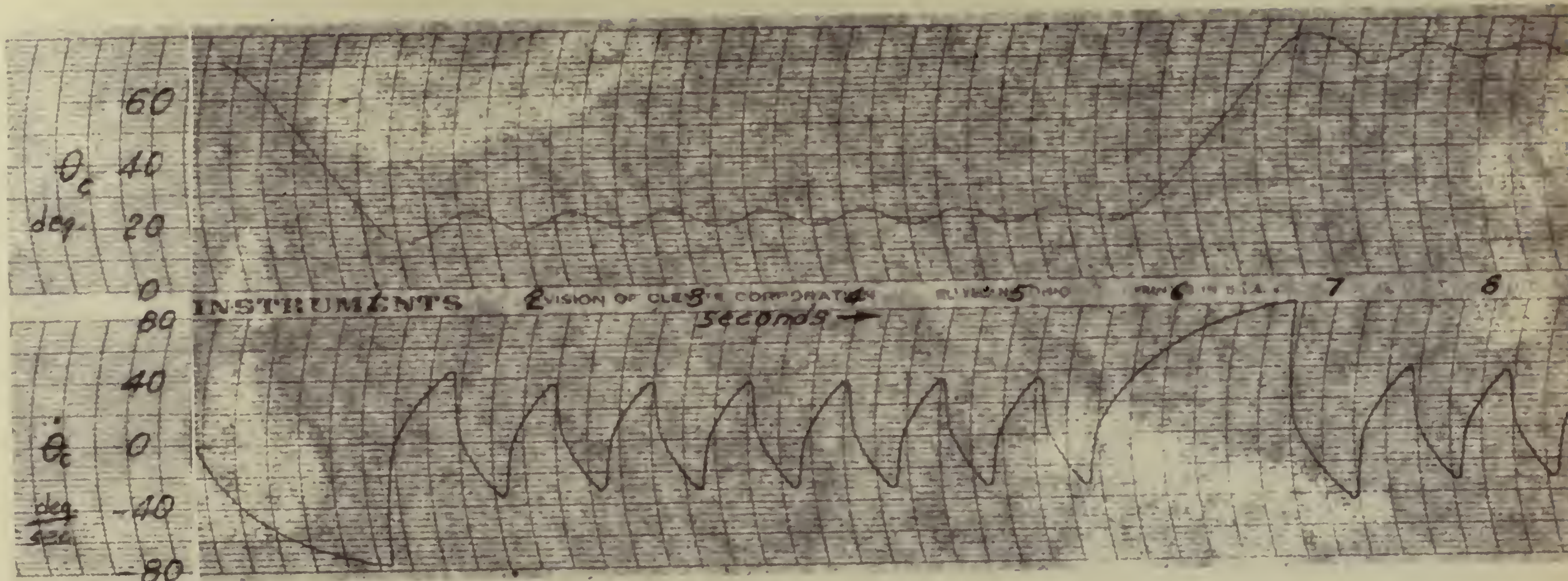

Fig. 8

SYSTEM RESPONSE TO ERROR CONTROL

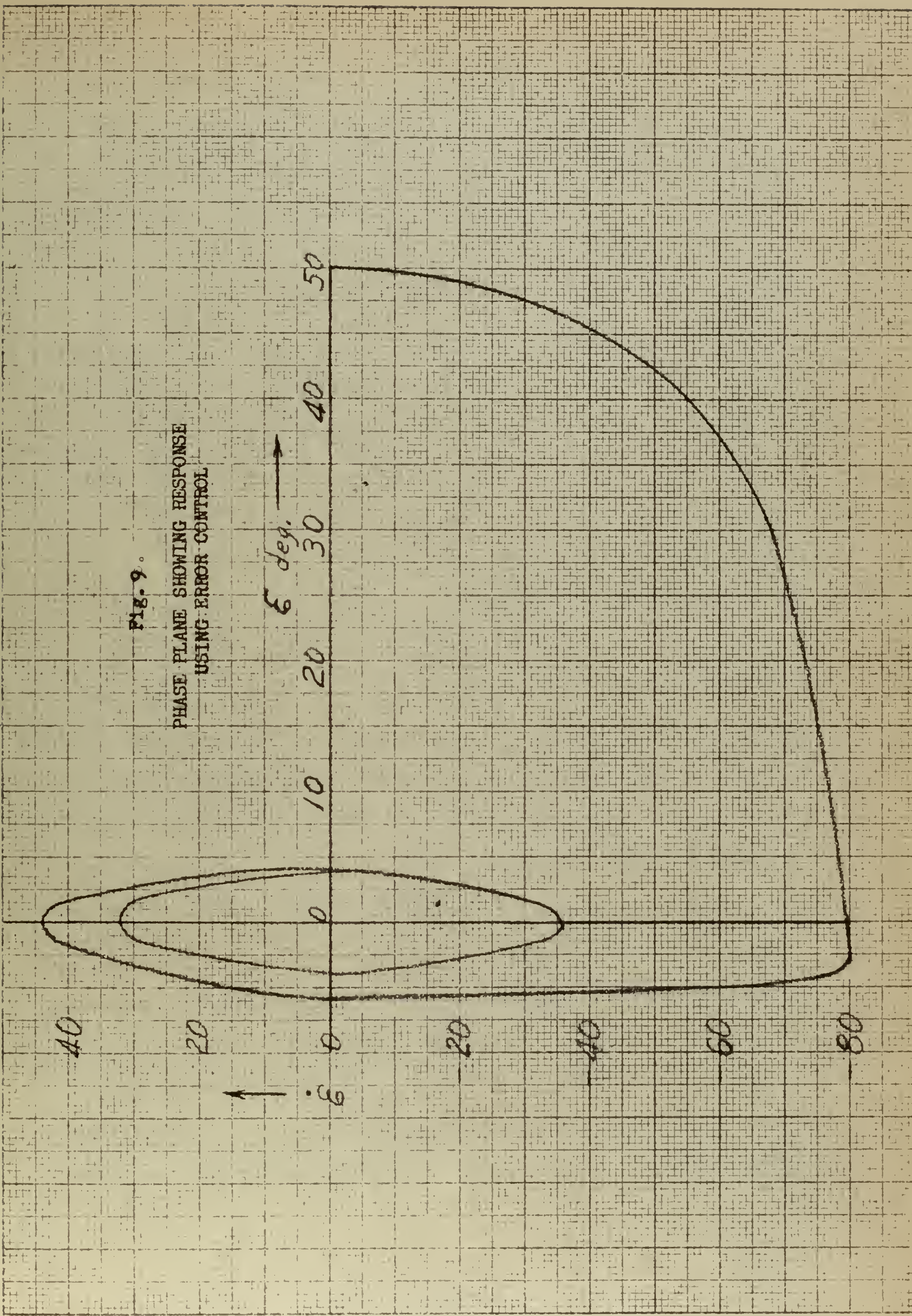

Fig. 9.

PHASE PLANE SHOWING RESPONSE USING ERROR CONTROL

cycle of constant magnitude about the origin of the phase plane. The velocity trace on Fig. 8 clearly shows the increased torque during deceleration due to the back electromotive force adding to the motor supply voltage during this phase of operation.

3. System Response Using Error and Error Rate Control.

If now a small derivative error feedback is added to the system, the relay switching line is rotated counterclockwise about the origin of the phase plane, giving the system a stabilizing effect. Relay reversal and the resultant deceleration torque now occur while the system is still approaching the commanded position.

The amount of error rate feedback is perhaps most conveniently expressed as the ratio of error to error rate used for relay control, since this ratio is the slope of the switching line from the vertical axis of the phase plane. Fig. 10 shows the response of the system to increasing amounts of error rate feedback. Note that as error rate feedback is increased, the limit cycle increases in frequency and decreases in amplitude. As error rate feedback is increased further, a limit cycle no longer exists, the response being a damped oscillation about the commanded position as shown in Fig. 11. Further increase in the $\dot{E}/E$ ratio increases the damping, thus reducing the number of oscillations, until, as the switching line coincides with the deceleration trajectory of the system, the response becomes dead beat as shown in Fig. 12. The system is now at near optimum response, the motor having accelerated under full forward torque to the switching line, then while still approaching the commanded position, it has decelerated under full reverse torque until the velocity reached zero and the error was within

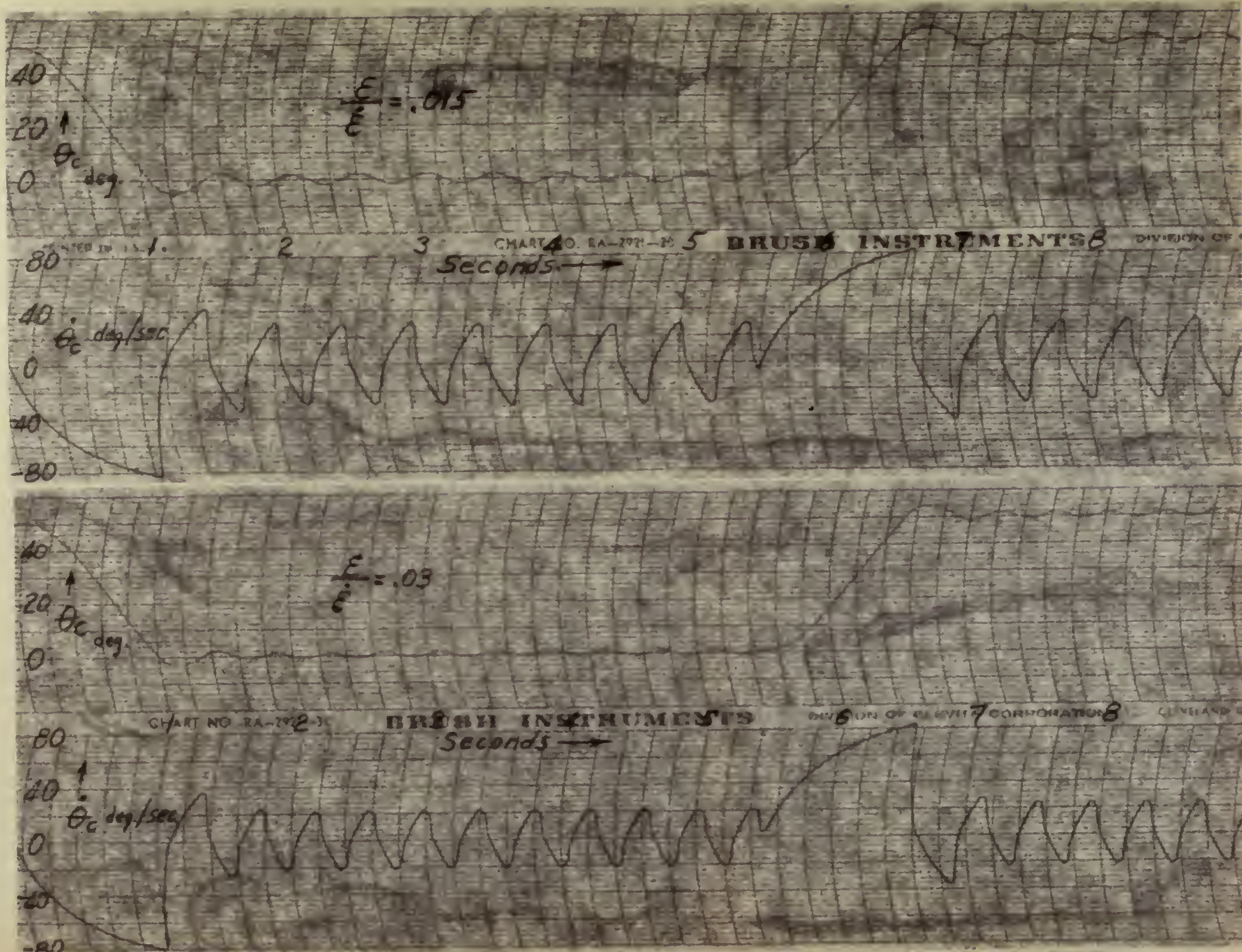

Fig. 10 RESPONSE TO INCREASING AMOUNTS OF ERROR RATE FEEDBACK

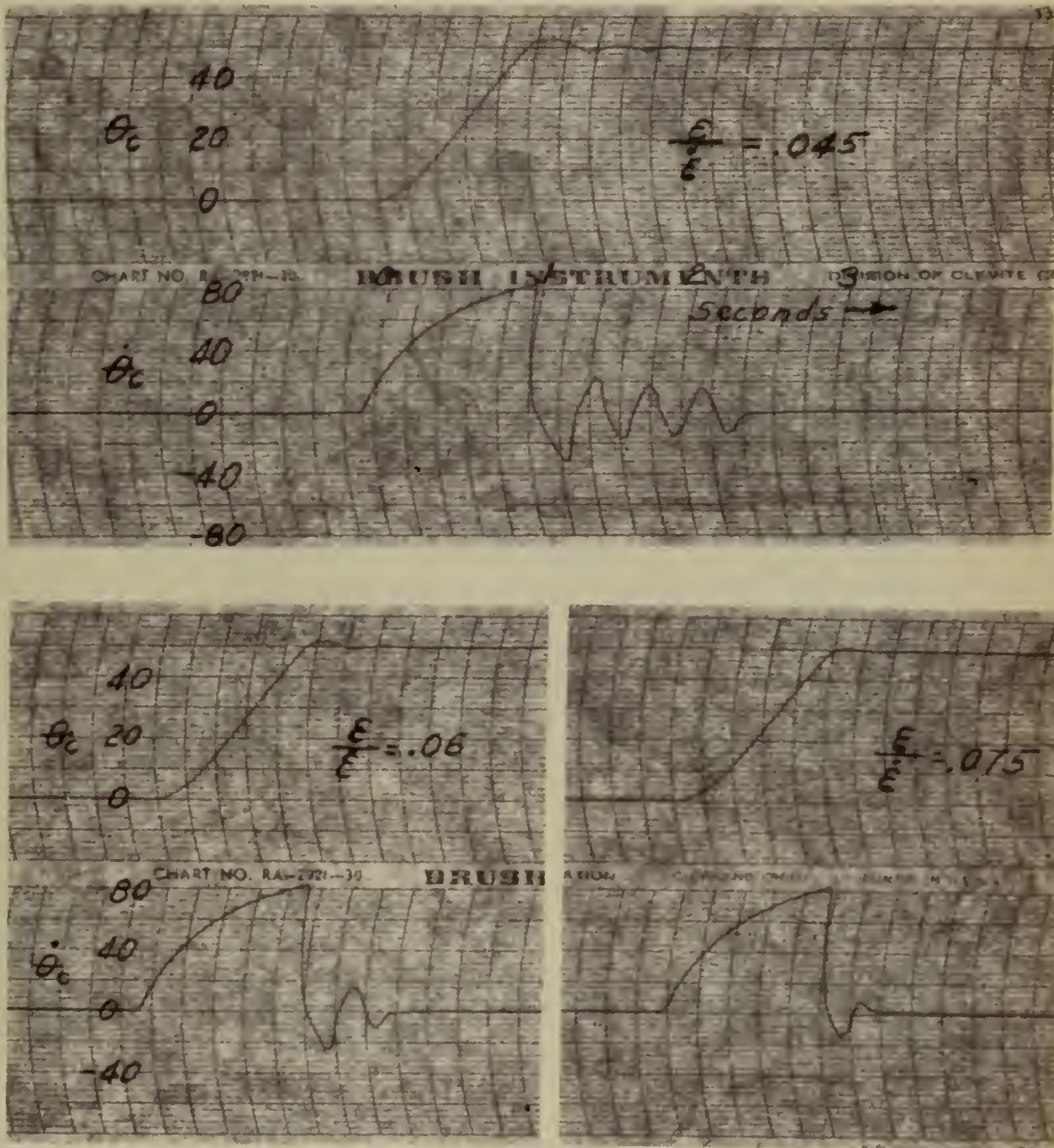

Fig. 11 RESPONSE TO FURTHER INCREASED ERROR RATE FEEDBACK

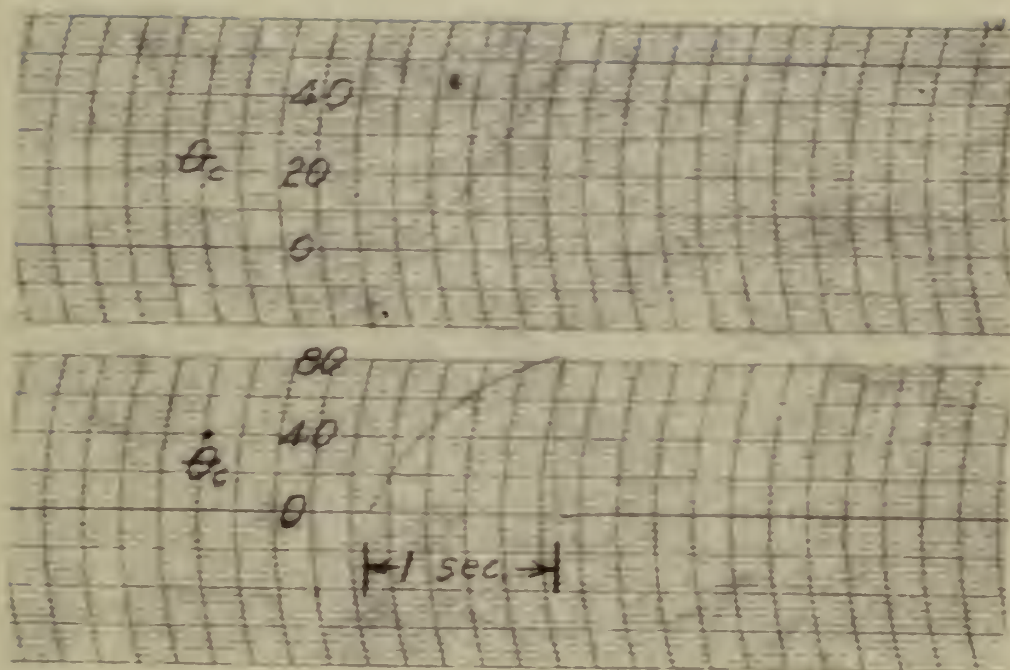

Fig. 12 DEAD BEAT RESPONSE OBTAINED WHEN RELAY SWITCHING LINE COINCIDES WITH DECELERATION TRAJECTORY OF SYSTEM

the dead zone at which time the relay opened. With a dead zone on the order of one degree on the systems tested the response was dead beat, there being neither relay stepping nor overshoot. The departure from optimum response was that the final output position lay between the reverse drop-out point and the forward pull-in point of the relay switching characteristic. Fig. 13 is a phase plane plot of system response with the Oster motor and a one degree relay dead zone. The solid curves show operation with 24 volts d. c. applied to the motor while the dotted curves show the response to identical step inputs with the motor operating on 28 volts RMS 60 cps power. Note that while an $\dot{E}/E$ ratio of .085 gave dead beat response in the case of d. c. operation, the ratio had to be increased to .094 to give dead beat response when operating on alternating current, due to the reduced torque developed in the latter case.

The fortunate result of having a deceleration trajectory which approximates a straight line is that the same $E/\dot{E}$ ratio is required for dead beat response to a small step displacement input as for a large one. Figs. 14 and 15 show phase plane plots of the Oster motor response to input steps ranging from 1.6 degrees to 296 degrees while using the same $\dot{E}/E$ ratio. Since velocity saturation was reached in the case of the 296 degree step, it is apparent that the dead beat operation will be obtained for any larger step, the limiting factor being only the physical characteristics of the error detector.

Fig. 16 shows the phase plane response of the Oster motor to steps of 10, 20, 30, and 40 degrees while operating on a motor supply voltage of 30 volts d. c.

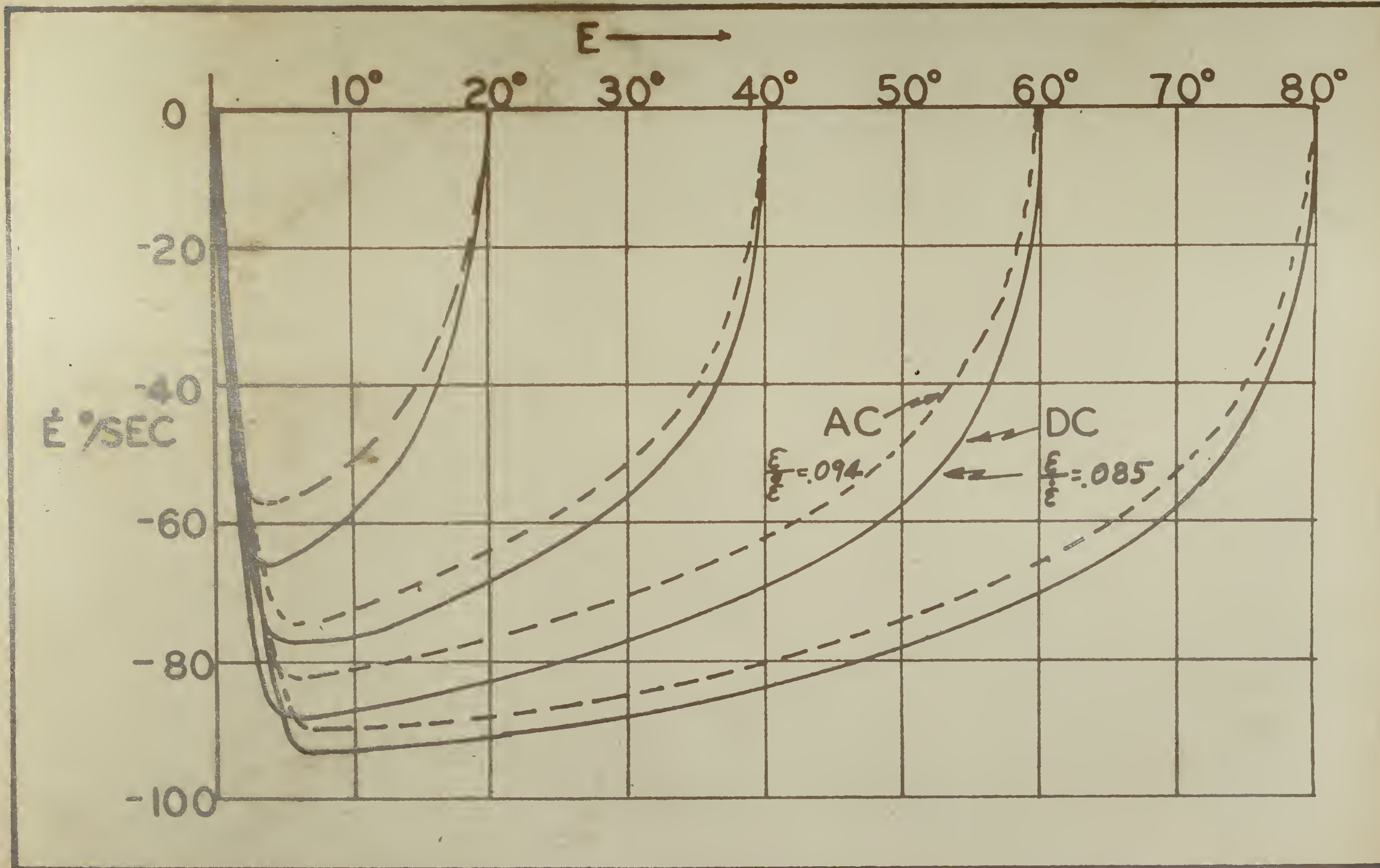

Fig. 13 PHASE PLANE SHOWING OPTIMUM RESPONSE USING THE OSTER MOTOR

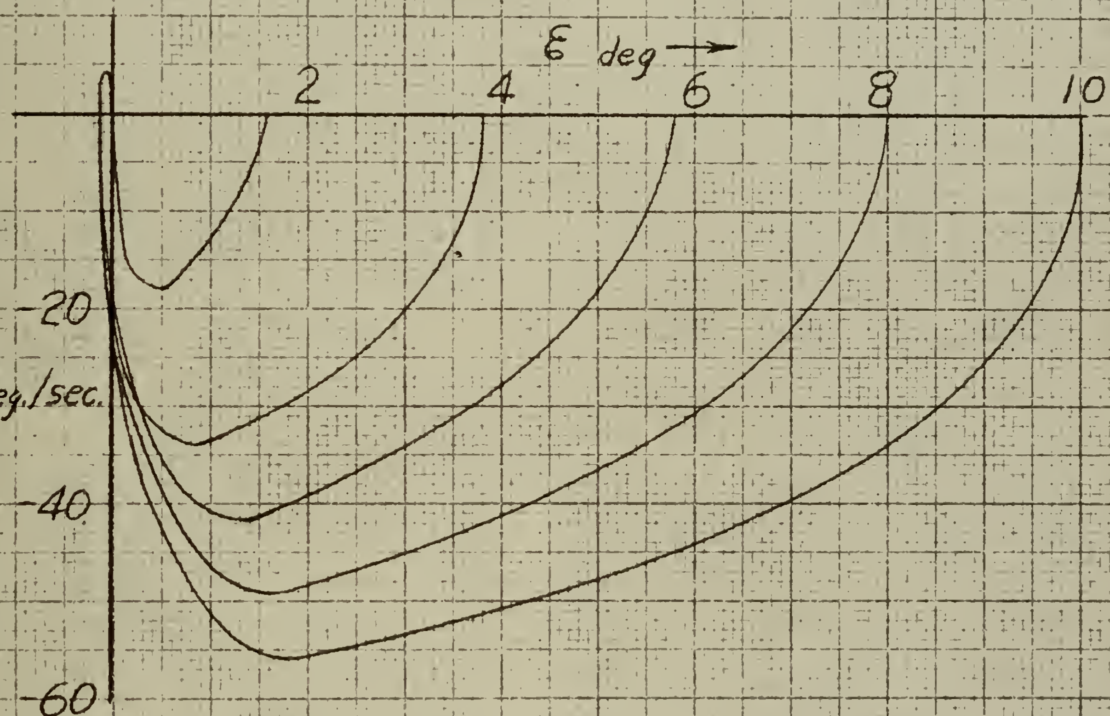

Fig. 14 RESPONSE TO SMALL STEPS

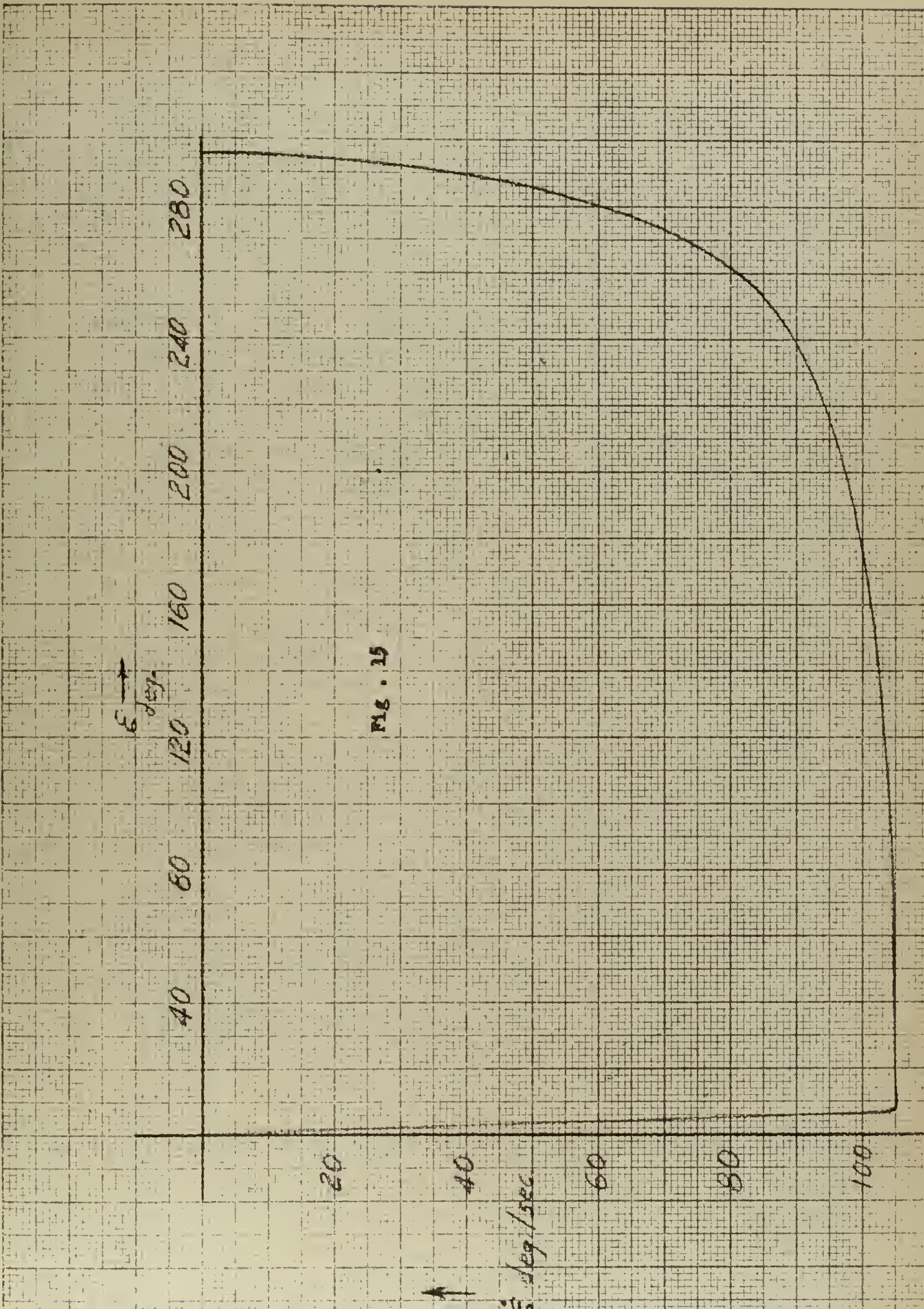

Fig. 15

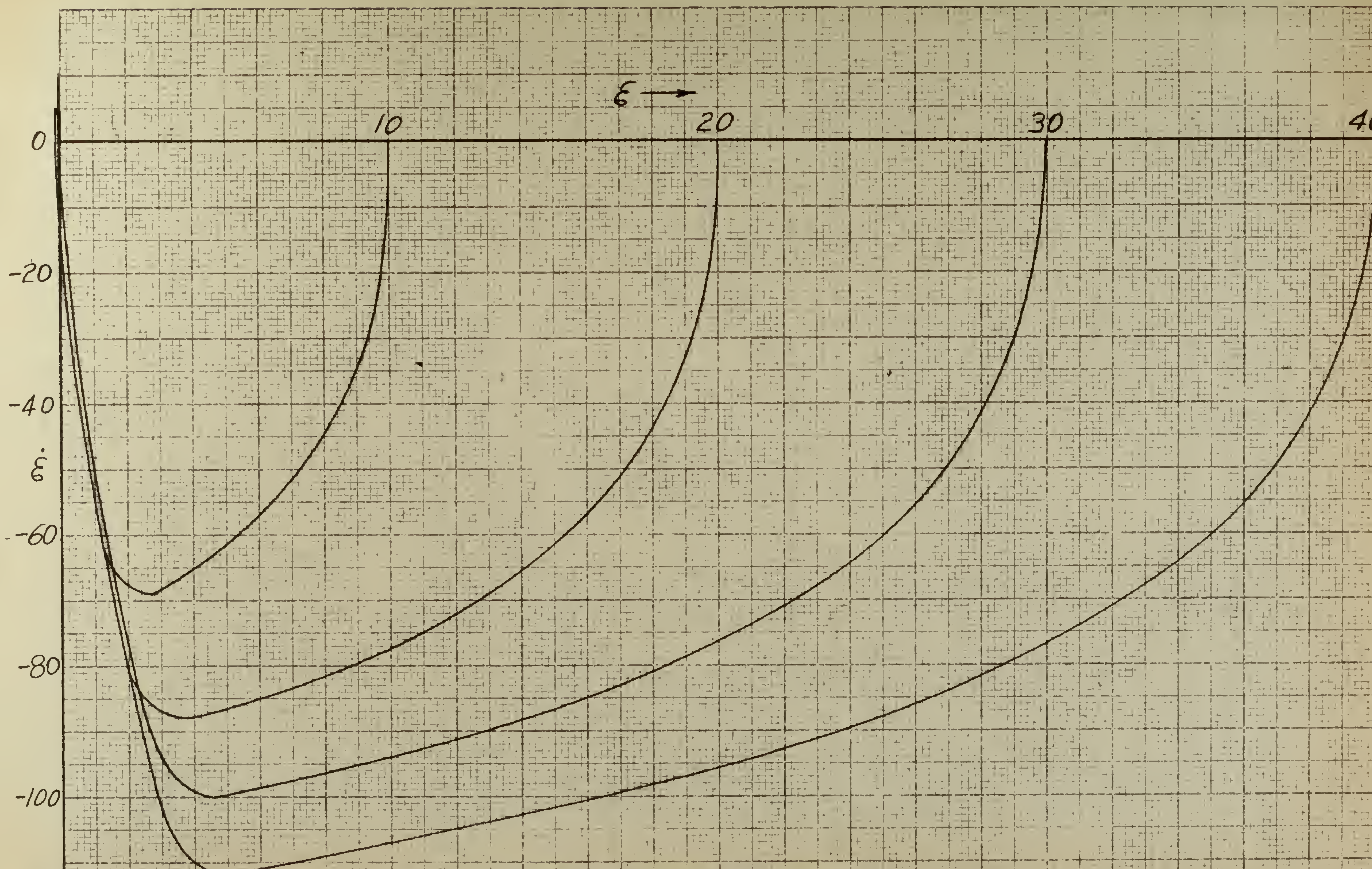

Fig. 16 RESPONSE OF OSTER MOTOR USING 30 VOLTS D. C.

Fig. 17 shows the response of the Elinco motor to various step inputs while operating on 60 volts d. c. and Fig. 18 shows the same dead beat response for the Elinco motor when operating on 83 volts RMS 60 cycle alternating current. Fig. 19 shows the response of the Marathon motor operating at 36 volts d. c. with an $E/\dot{E}$ ratio slightly too small for dead beat response. The result is a position overshoot of 0.8 degrees in the case of the larger steps.

The effect of increasing the $E/\dot{E}$ ratio beyond that required for dead beat response is to cause the system to step forward one or more times before reaching the commanded position, the number of forward steps depending upon the amount of excess error rate feedback applied. A transient record of position and velocity of the Oster motor with excess error rate feedback is shown in Fig. 20. The required $E/\dot{E}$ ratio for dead beat response in this case was .085. The two upper curves show a single forward step and were recorded while using $E/\dot{E}$ ratios of .10 and .12. The lower curve of Fig. 20 shows three forward steps which resulted when the $E/\dot{E}$ ratio was increased to .15.

4. System Response With Very Narrow Dead Zone.

In order to determine the suitability of the series motor in relay servos where high positioning accuracy is required at the expense of dead beat response, the relay dead zone was reduced to 0.1 degrees and transient records made of position and velocity using the Oster motor with 24 volts d. c. supply voltage. The response was not dead beat, the system either overshooting a small amount in the case of switching too late, or stepping forward in the case of switching too early. In the former case, the deceleration curve crosses the horizontal

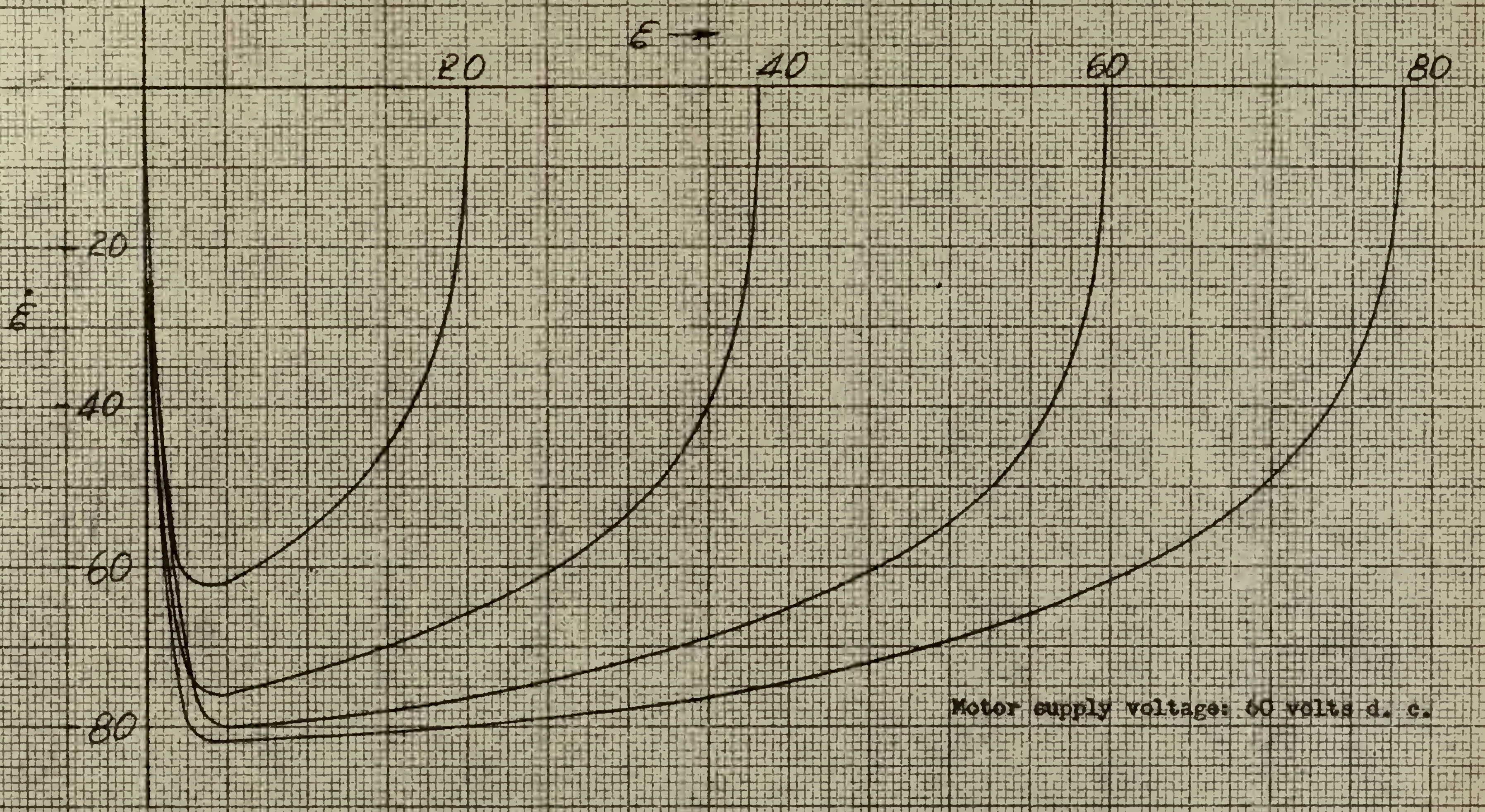

Fig. 17 PHASE PLANE SHOWING OPTIMUM RESPONSE OF ELINCO MOTOR

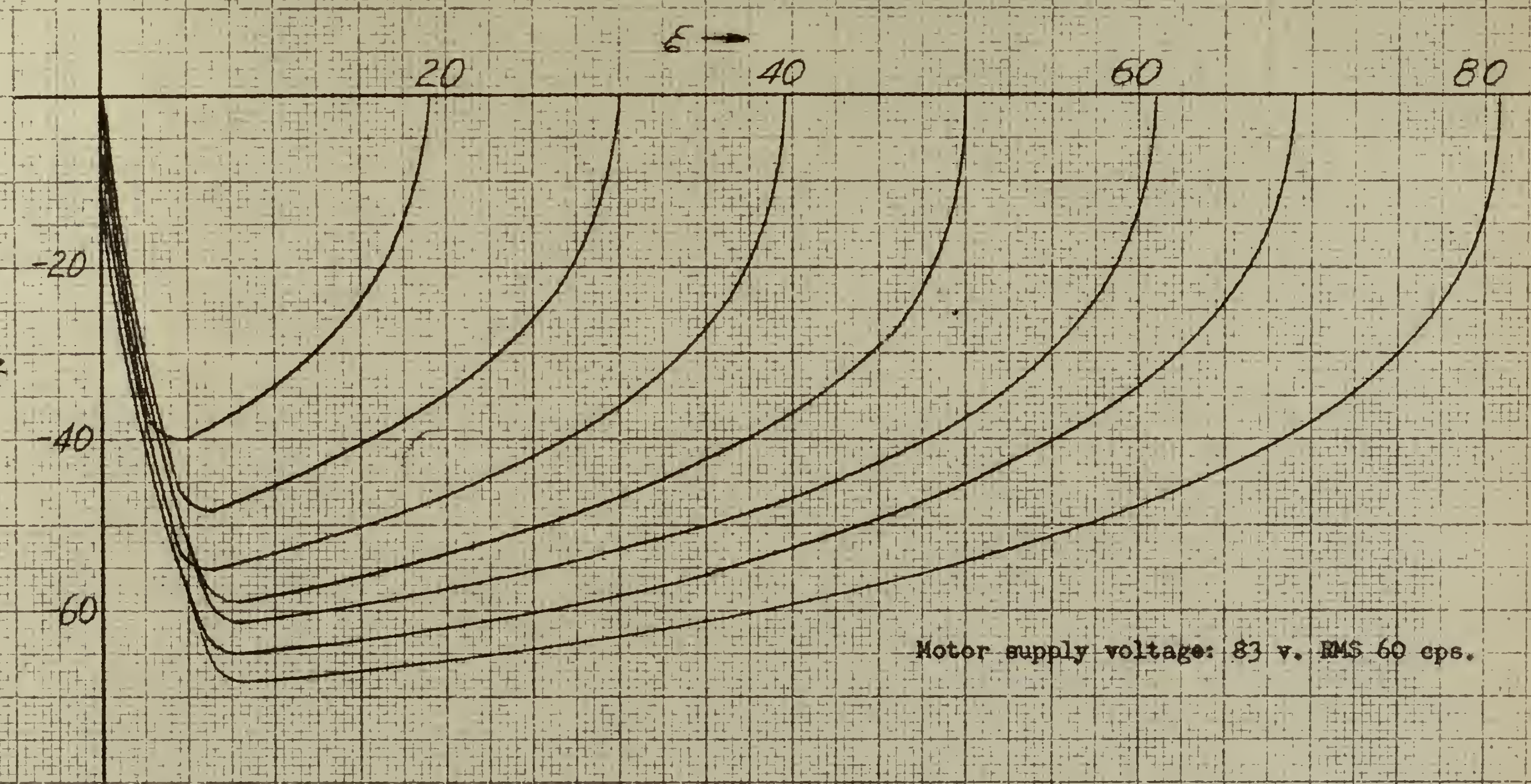

Fig. 18 PHASE PLANE SHOWING OPTIMUM RESPONSE OF ELINCO MOTOR OPERATING ON A. C.

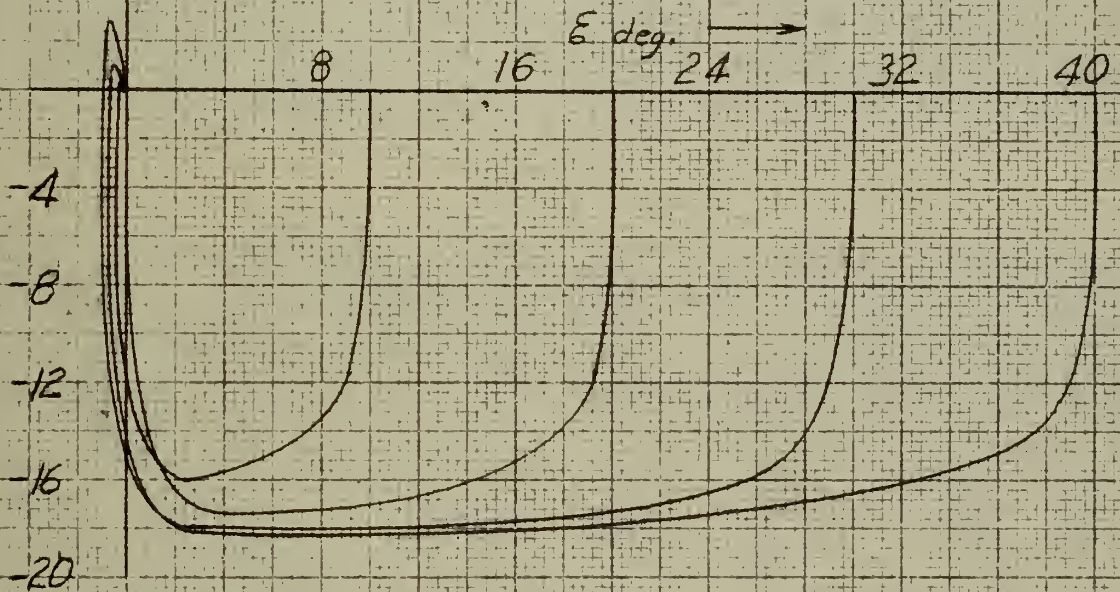

Fig. 19 PHASE PLANE SHOWING RESPONSE OF MARATHON MOTOR TO STEP DISPLACEMENT INPUTS

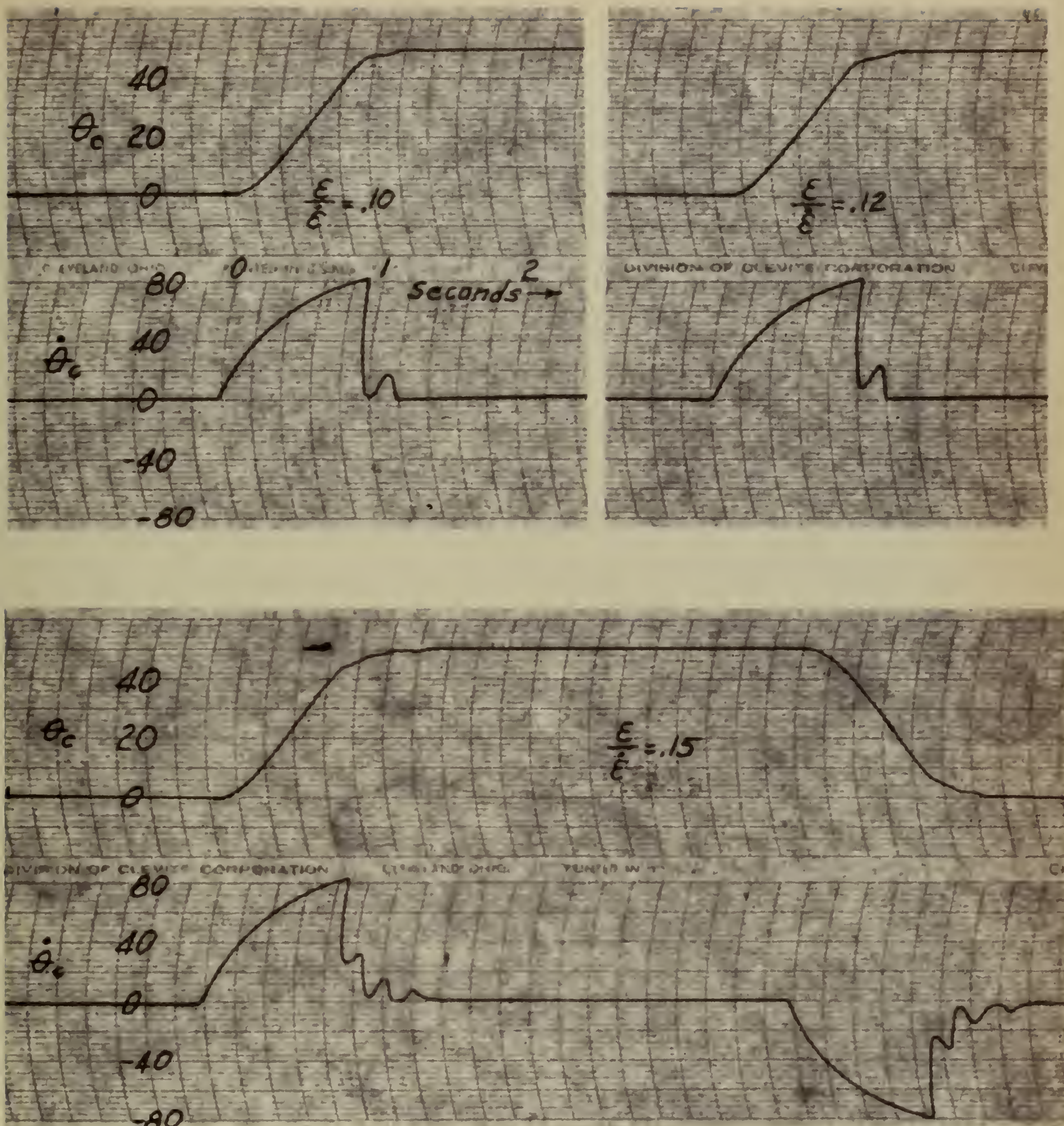

Fig. 20. FORWARD STEPPING OPERATION WHICH OCCURS WHEN E/Ė RATIO IS INCREASED BEYOND THE VALUE REQUIRED FOR DEAD BEAT RESPONSE

axis of the phase plane and there is a velocity reversal prior to reaching the relay reverse drop out line. The relay then opens and the system comes to rest within the dead zone due to friction. In the latter case the system steps forward into the dead zone due to the curvature present in the deceleration trajectory. Either type of response may be selected by adjusting the $E/\dot{E}$ ratio used for control. Records of transient position and velocity using the 0.1 degree relay dead zone are presented in Figs. 21 and 22. Conditions of overshoot and forward stepping are both shown. Note that either response is virtually indistinguishable from dead beat performance on the position trace, it being necessary to refer to the velocity trace to determine which condition prevails.

Fig. 23 shows a sketch of a typical deceleration trajectory when the relay dead zone is on the order of one degree and the response is dead beat. Note that the relay switching lines are essentially parallel to the straight line portion of the deceleration trajectory. This is achieved experimentally by adjusting the $E/\dot{E}$ ratio to obtain dead beat response. Deceleration takes place under full reverse input to the motor. Just prior to reaching the horizontal axis, the deceleration curve becomes vertical, crossing the reverse drop out line of the relay, and the velocity reaches zero with the position between the reverse drop out line and forward pull in line of the relay switching characteristic.

Fig. 24 shows a sketch of the deceleration trajectory obtained with narrow dead zones of 0.5 degrees or less. Here the response is not dead beat, the system either overshooting a small amount in the case of switching too late, or stepping forward in the case of switching too early.

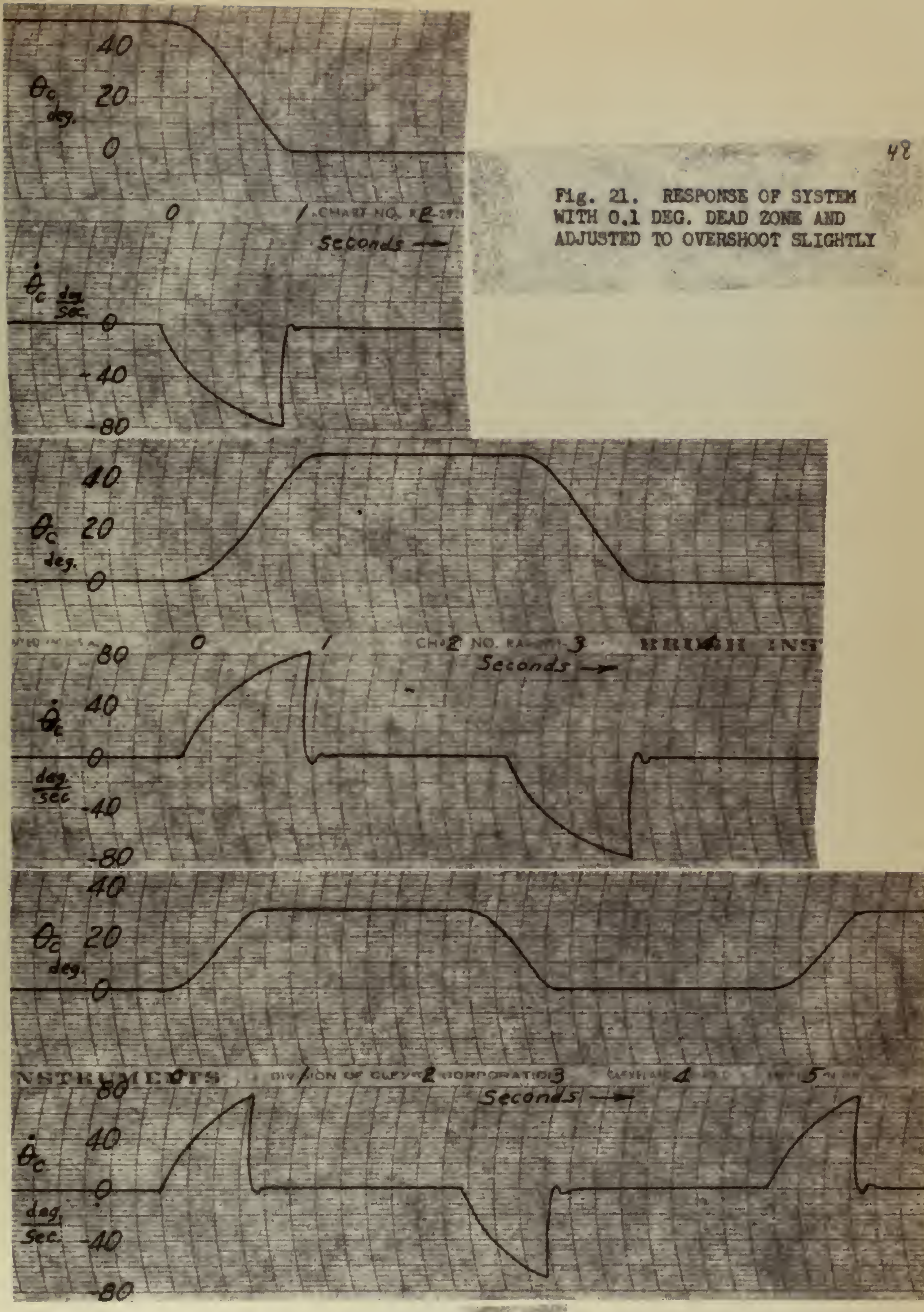

Fig. 21. RESPONSE OF SYSTEM WITH 0.1 DEG. DEAD ZONE AND ADJUSTED TO OVERSHOOT SLIGHTLY

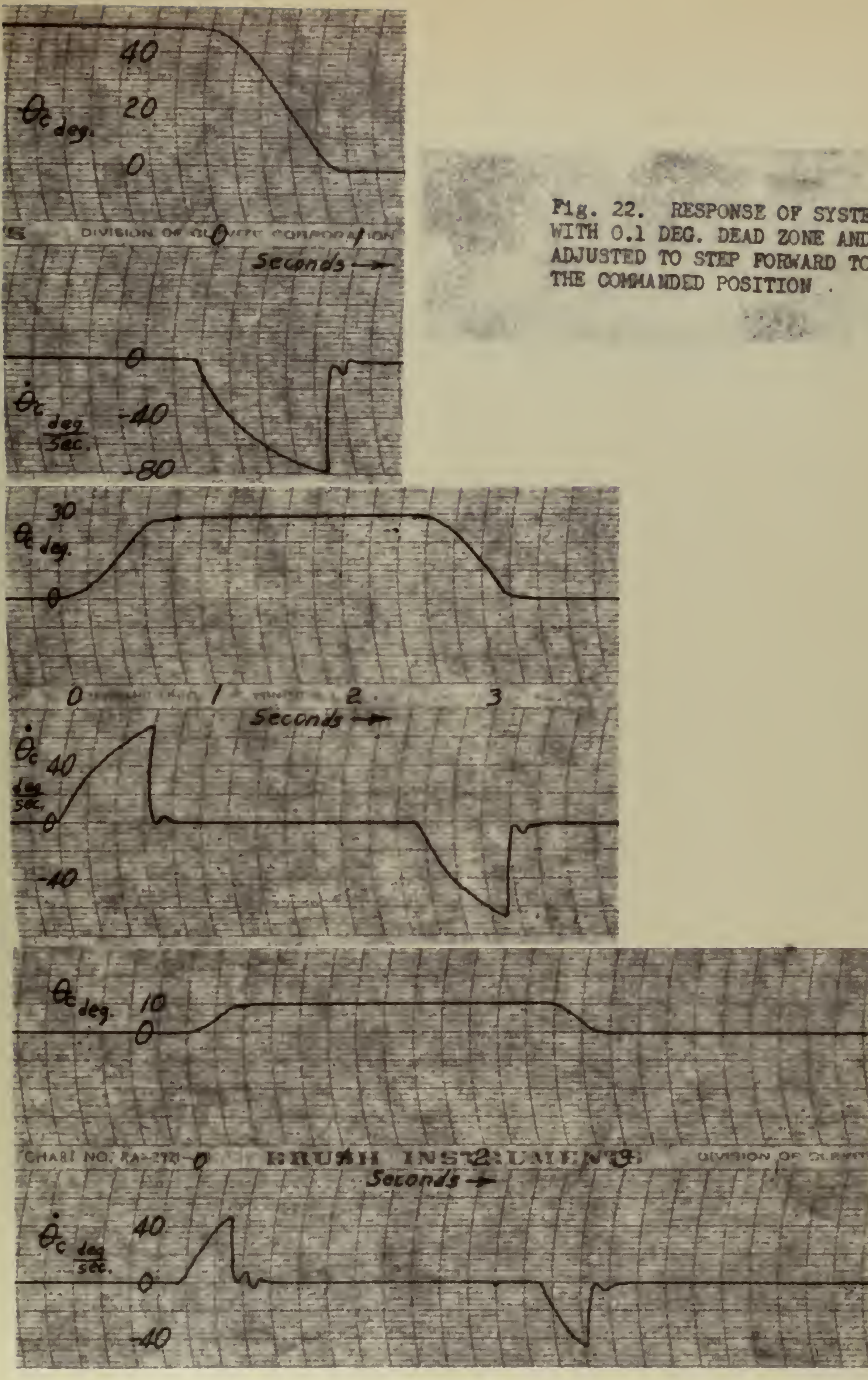

Fig. 22. RESPONSE OF SYSTEM WITH 0.1 DEG. DEAD ZONE AND ADJUSTED TO STEP FORWARD TO THE COMMANDED POSITION .

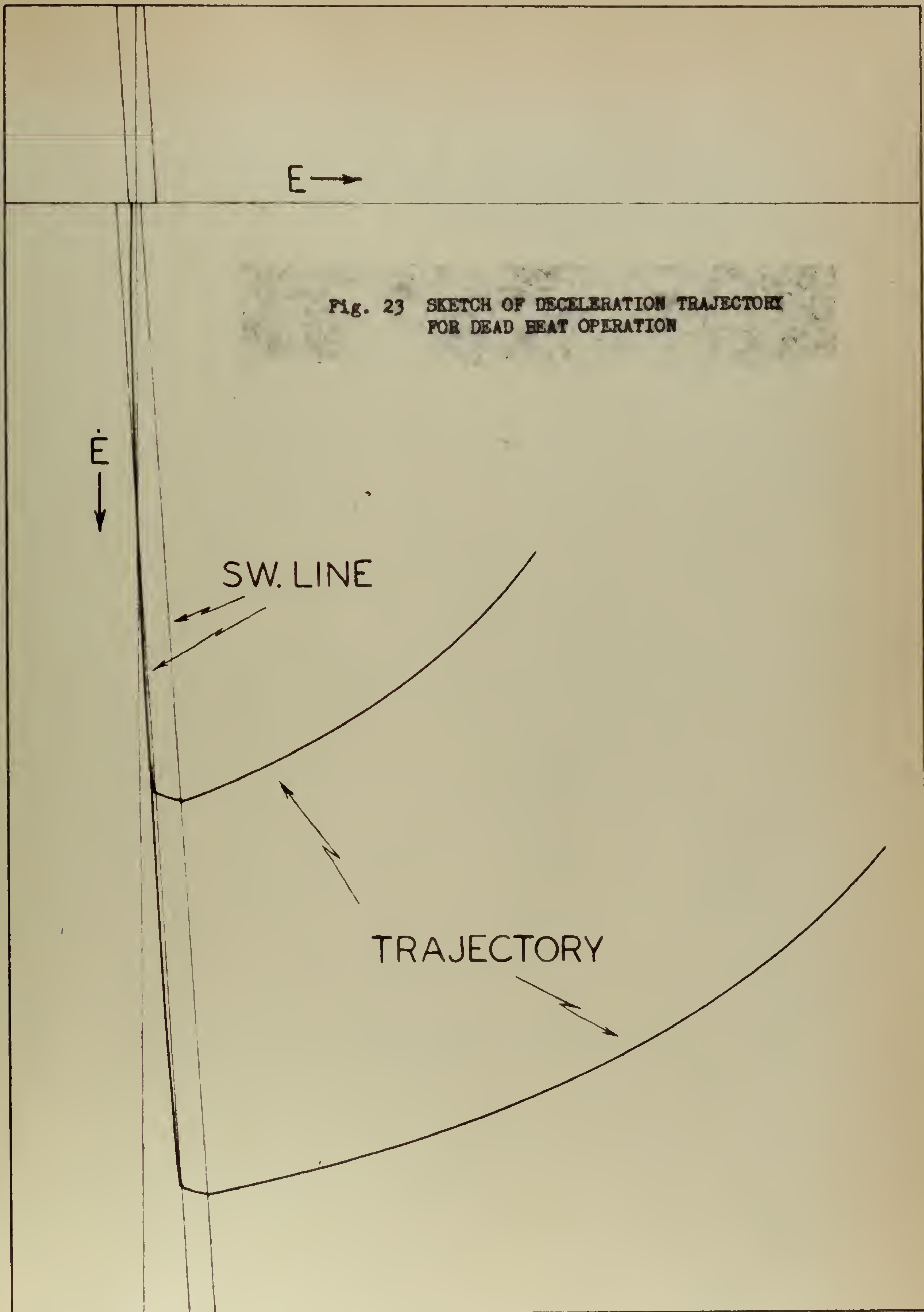

Fig. 23 SKETCH OF DECELERATION TRAJECTORY FOR DEAD BEAT OPERATION

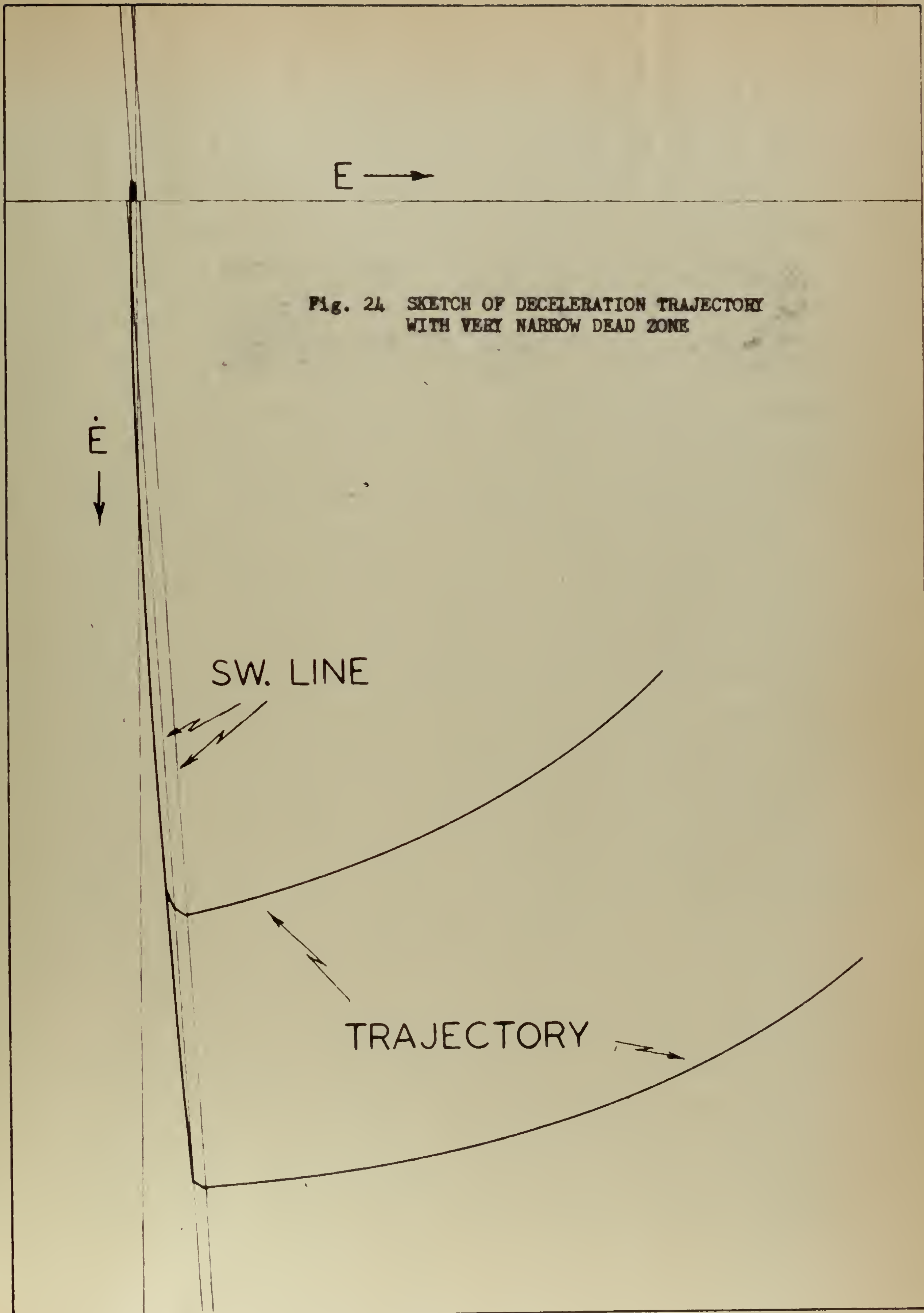

Fig. 24 SKETCH OF DECELERATION TRAJECTORY
WITH VERY NARROW DEAD ZONE

In the former case the deceleration curve crosses the horizontal axis and there is a velocity reversal prior to reaching the relay reverse drop out line. The relay then opens and the system comes to rest within the dead zone due to friction. In the latter case the system steps forward into the dead zone due to the curvature present in the deceleration trajectory.

5. Motor Operation Using Alternating Current.

While the three motors tested were designed for d. c. operation, tests were conducted on the Elinco and the Oster motors using 60 cycle supply and on the Oster using 400 cycle supply to determine whether or not the dead beat response would be obtained for a series motor under these conditions. In the case of a. c. operation, supply voltages were used which gave rated motor speed in the steady state condition. Operation on a. c. resulted in considerably lower acceleration and deceleration torques than were obtained with direct current. Dead beat response to any size step input was obtained in all three cases, however, by increasing the $E/\dot{E}$ ratio the required amount to match the relay switching line with the deceleration trajectory of the system for the particular supply voltage selected. Figs. 4, 13, and 18 show the system response while operating on alternating current.

6. Inertia Effects.

The effect of inertia in the system of course lowers the acceleration and deceleration. To determine whether dead beat response to any size step input would be obtained for various amounts of fixed system inertia, inertia discs were added, increasing the inertia. Here again, after increasing the $E/\dot{E}$ ratio to the proper point, dead beat response

was obtained for all step displacement inputs. Fig. 25 shows the system response to step inputs with the increased inertia.

7. Response To Small Steps and the Effect of Varying Motor Supply Voltage.

When methods of optimum relay servo theory are applied to a servo to obtain dead beat response, difficulty is usually encountered in the response to small steps. This problem can be serious enough as to require a dual mode system wherein linear operation is obtained near the origin of the phase plane. It is here that the series motor appears to present a great advantage since it's response remains dead beat for any size step displacement input. Moreover, the dead beat operation holds for different supply voltages to the motor, once the proper switching line is located for the particular motor voltage selected. An increase in motor supply voltage is accompanied by a steeper deceleration trajectory, thus a lower ratio of $E/\dot{E}$ for control is required as the supply voltage is increased. The effect of poor regulation in the motor supply voltage source is to reduce the slope of the deceleration trajectory, thus requiring relay reversal at an earlier time for dead beat response. This condition is met by selecting a greater $E/\dot{E}$ ratio for control, thus rotating the switching line counterclockwise to the required position.

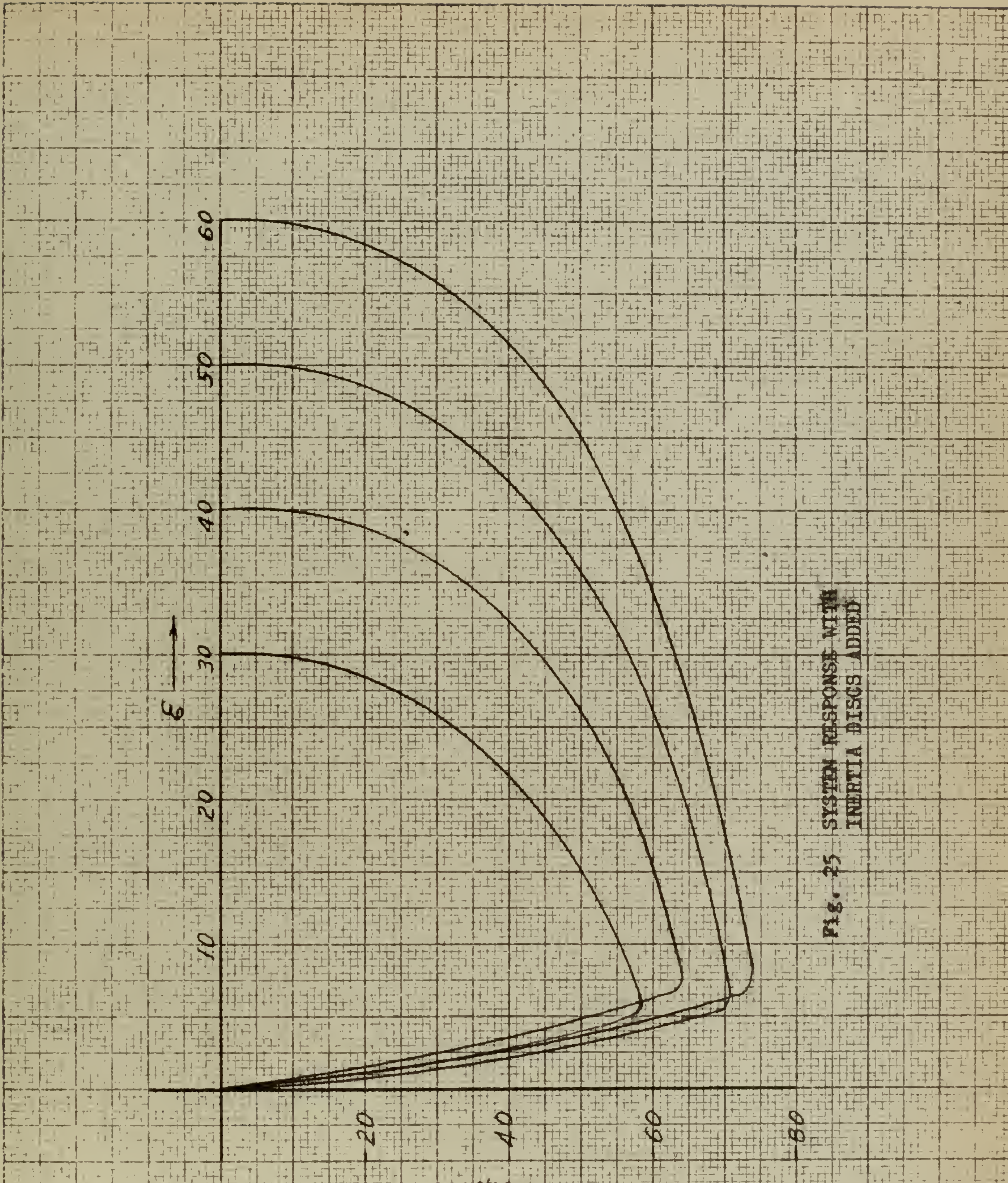

Fig. 25 SYSTEM RESPONSE WITH INERTIA DISCS ADDED

RESPONSE TO RAMP INPUTS

A relay servo is not normally used to follow ramp inputs since the discontinuous nature of the system will always cause a limit cycle for this type of input. In addition, if tachometer feedback is used a steady state error is introduced, much the same as the velocity lag error encountered with a linear servo following a ramp input.

In order to determine the response of the optimized series motor system to a ramp input, tests were conducted using the Oster motor with ramp inputs ranging from ω_R = 1.5 deg./sec. to ω_R = 19.25 deg./sec. Fig. 26 shows a Brush Recorder tape of θ_R and E for a ramp input. The tachometer feedback used was that required for dead beat transient response. This tachometer feedback, in the case of the series motor, is small enough to permit ramp input operation with reasonable steady state errors. The relay operation for all ramp inputs was that of stepping forward.

Steady state error to ramp inputs is herein defined as the average error, or center of the limit cycle. It varied from 0.5 degrees (the dead zone width in this case) for a ramp input velocity of 1.5 deg./sec. up to 2.8 degrees for an input velocity of 19.25 deg./sec. The peak to peak amplitude of the limit cycle varied from 0.3 degrees for the 1.5 deg./sec. input up to 0.7 degrees in the case of the 19.25 deg./sec. input. Fig. 27 shows a plot of steady state error and peak to peak amplitude of the limit cycle versus ramp input velocity.

If ramp input velocities greater than the above are to be encountered, or if it is necessary to reduce the steady state error to ramp inputs, it should prove desirable to use error differentiation or phase lead

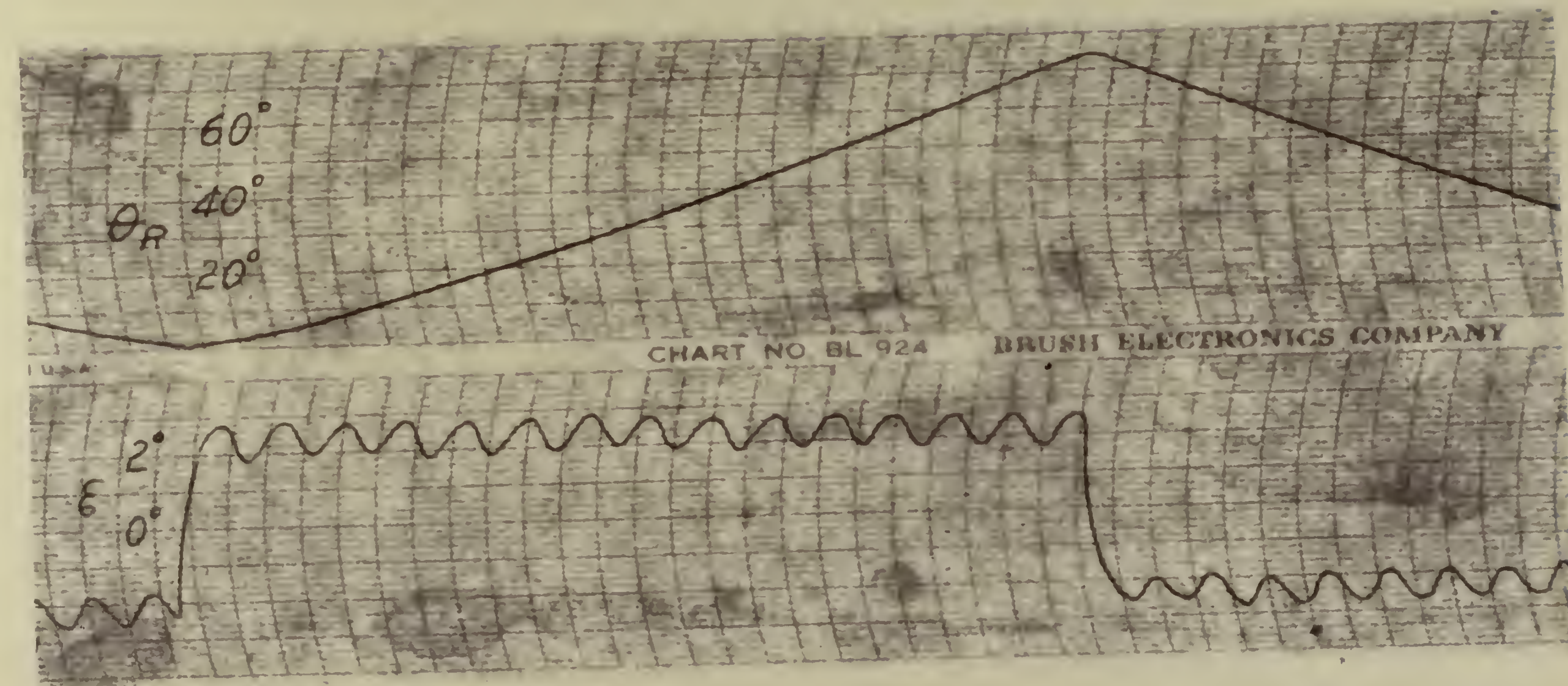

Fig. 26 θ_R AND E FOR A RAMP INPUT

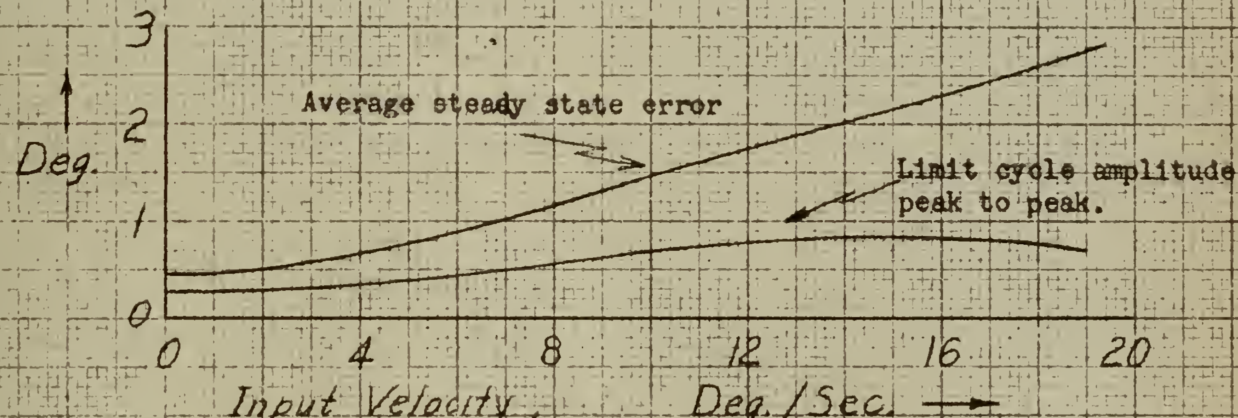

Fig. 27 STEADY STATE ERROR AND PEAK TO PEAK AMPLITUDE
OF THE LIMIT CYCLE VERSUS RAMP INPUT VELOCITY

compensation rather than tachometer feedback for system stabilization. There is no reason why this should detract from the transient response characteristics.

FREQUENCY RESPONSE

The system response was then determined for sinusoidal inputs. Here again the $\dot{E}$ feedback used was that required for dead beat transient response. Simultaneous Brush tapes of θ_R and θ_C were recorded. Sinusoidal inputs of 20, 40, 60, and 80 degrees peak to peak were applied at frequencies from 0.1 cps to 2.0 cps. Fig. 28 shows typical records of θ_R and θ_C versus time. Magnitude and phase angle data were measured directly from the tapes and plotted on Fig. 29 and Fig. 30 respectively. These data are not precise in that some error due to harmonics is present. For more precise magnitude and phase data, the Fourier Series fundamental of the output waveform would have to be determined for each amplitude and frequency applied. Harmonic distortion of the output wave is not great, however, as can be seen in Fig. 28, therefore, the curves of Figs. 29 and 30 should not be greatly in error.

Fig. 29 shows a very interesting characteristic of the optimized system; the absence of overshoot in following any sinusoidal input. The response is flat at unity, then falls off sharply. Cut off occurs at a lower frequency and is more sharply defined as the magnitude of the signal input is increased. This frequency response may prove useful where system bandwidth considerations are important.

Fig. 28 θ_R AND θ_C VERSUS TIME FOR SINUSOIDAL INPUTS

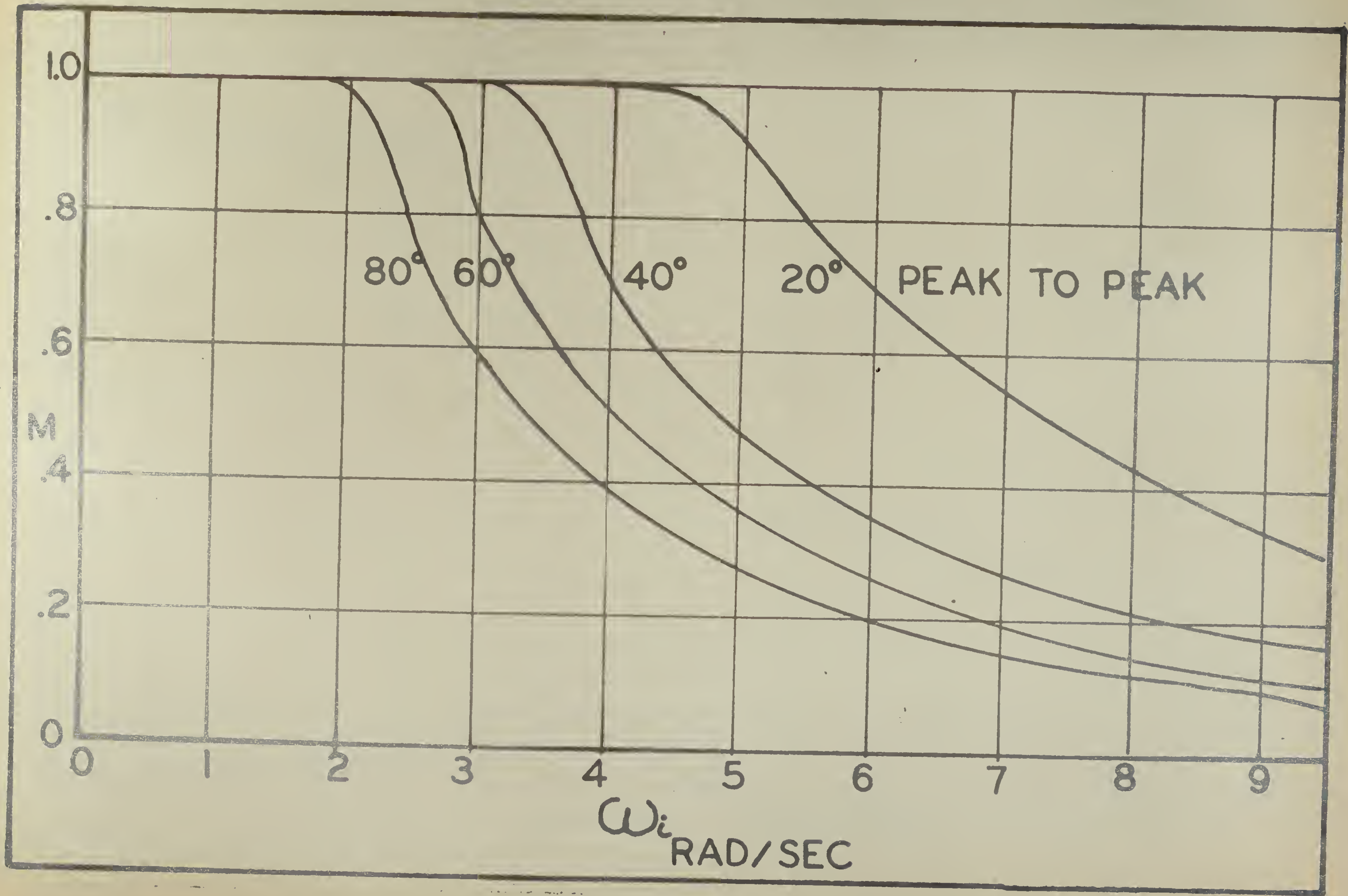

Fig. 29 FREQUENCY RESPONSE--MAGNITUDE RATIO

TOP

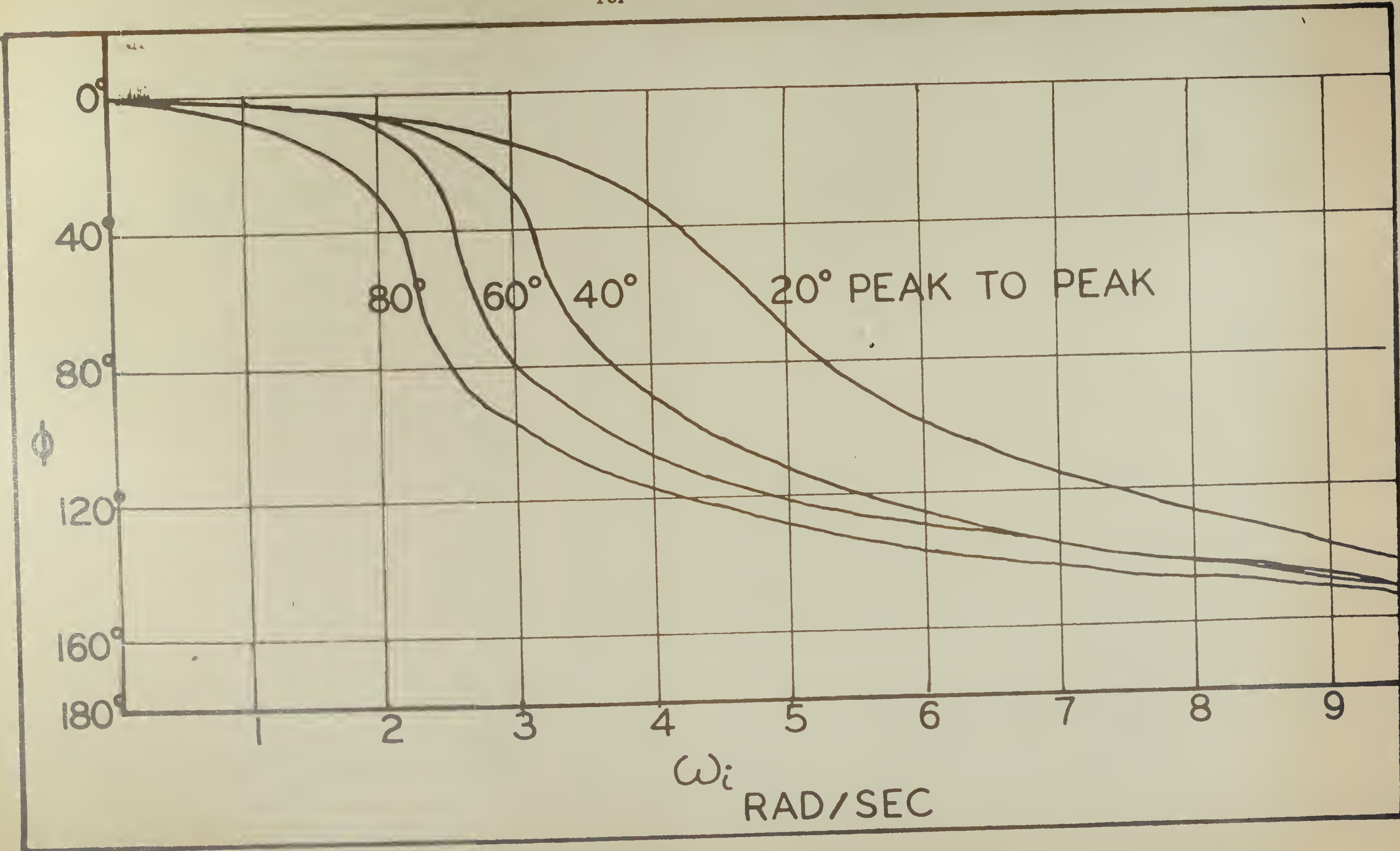

Fig. 30 FREQUENCY RESPONSE--PHASE ANGLE

CONCLUSIONS AND RECOMMENDATIONS

It is concluded that:

1. The split field series motor, with it's current squared Torque characteristic, provides an ideal method of driving a relay operated servomechanism where fast response is required.

2. The deceleration torque of the series motor is such as to permit dead beat operation for any size step displacement input by the use of a linear switching line in the phase plane. Such operation varies from true optimum response only in that a small static error is present due to the finite relay dead zone.

3. Linear relay switching criteria, being a function of error and error rate, are readily obtained by summing error and tachometer signals.

4. Dead beat response to any size step displacement input was obtained from all three motors with relay dead zones of 0.5 degrees or greater.

5. Dead beat response was obtained using various fixed motor supply voltages, d. c. or a. c., and for any fixed inertia loading desired.

6. System operation was satisfactory with a relay dead zone as small as 3 minutes of arc so long as true dead beat operation was not required. Even under these conditions departure from dead beat operation was not observable on the position versus time curve, only on the velocity versus time curve.

7. The relay operated series motor servo, adjusted for dead beat response to step displacement inputs, shows reasonably good response to ramp inputs.

8. Response of the optimized system to sinusoidal inputs is remarkably flat over a wide frequency range, after which it falls off sharply.

It is recommended that:

1. Further study be made of the relay controlled series motor servomechanism using the silicon controlled rectifier in lieu of the electromechanical relay for motor control. This device should provide switching characteristics approaching those of the ideal relay in that relay time lag could be reduced and the dead zone could be made as small as desired.

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2. Goslow, Paul; "Investigation of a Relay Servo Using a Series Motor" (Unpublished Masters Thesis, U. S. Naval Postgraduate School, Monterey, California, 1958)